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EXISTENCE, UNIQUENESS AND LIMITING BEHAVIOR OF SOLUTIONS OF A CLASS OF DIFFERENTIAL EQUATIONS IN BANACH SPACE

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Let X be a Banach space (real or complex) and A_n and B be linear operators in X with $D(B) \subseteqq D(A_n)$, $n = 1, 2, \cdots$. The following note is concerned with existence and uniqueness of solutions of the problem

$$\frac{d}{dt}[(I - A_n)u(t)] - Bu(t) = 0, \quad (t > 0), \quad u(0) = u_0 , \tag{1.1}$$

and the limiting behavior of these solutions as the operators A_n tend to zero in a sense to be specified. We will show that for a large class of operators the problem (1.1) is well posed and that its solutions tend to the solution of the problem

$$\frac{du(t)}{dt} - Bu(t) = 0, \quad (t > 0), \quad u(0) = u_0 . \tag{1.2}$$

In particular, we obtain an extension to Banach spaces of a result of R. E. Showalter [5] to the effect that (1.1) is well posed when X is a Hilbert space and A_n and B are maximal dissipative operators in X which satisfy the algebraic condition

$$\operatorname{Re}((I - A_n)x, Bx) \leqq 0 , \qquad x \in D(B) \subseteqq D(A_n) . \tag{1.3}$$

In the next section we give sufficient conditions for (1.1) to be well posed. We note that these conditions do *not* guarantee that (1.2) is well posed. In § 3 we show that if, in addition, $\{A_n\}$ tends to zero in a certain sense, then (1.2) is well posed and the solutions u_n of (1.1) tend to the solution of (1.2). In particular, it will follow that if A and B are densely defined maximal dissipative operators in a Hilbert space and if (1.3) is satisfied with $A_n = n^{-1}A$, then

$$\frac{d}{dt}[(I - n^{-1}A)u_n(t)] - Bu_n(t) = 0 , \quad (t > 0) , \quad u_n(0) = u_n \in D(B) ,$$

is well posed and as $n \to \infty$, u_n converges strongly to the unique solution of (1.2). Two examples are discussed in § 4.

We emphasize that throughout this paper it is assumed that $D(B) \subseteqq D(A_n)$. The question of limiting behavior of solutions of (1.1) when X is a Hilbert space, $A_n = n^{-1}A$ and $D(A) \subseteqq D(B)$ has been considered previously [2], and it is interesting to compare the results of [2] with those of the present note in the case $D(A) = D(B)$. In [2] it was assumed that A and B were maximal dissipative operators