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FIXED POINTS AND CHARACTERIZATIONS OF CERTAIN MAPS

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Let T be a self map on a metric space (X, d) such that

$$d(T(x), T(y)) \leqq (d(x, T(x)) + d(y, T(y))/2 , \qquad x, y \in X .$$

It is proved that: (a) T has a fixed point if T is continuous and X is weakly compact convex subset of a Banach space. (b) All such T which have fixed points can be explicitly determined in terms of d. Related results are obtained.

1. **Introduction.** In [7], [8], [9], [10], [11], R. Kannan considered the following family of self maps T on a (nonempty) complete metric space (X, d):

$$(1) \qquad d(T(x), T(y)) \leqq \frac{1}{2}(d(x, T(x)) + d(y, T(y))) , \qquad x, y \in X .$$

He obtained a number of results of the following type: T has a (unique) fixed point if X is a weakly compact convex subset of a reflexive Banach space B and for each closed convex subset H of X with $T(H) \subset H$ and $\delta(H) > 0$,

$$(2) \qquad \sup \{d(y, T(y)): y \in H\} < \delta(H) ,$$

where d is the metric induced by the norm $\| \ \|$ on B and $\delta(H)$ is the diameter of H. Suppose now that X is a weakly compact convex subset of a Banach space B and T is a self map on X which satisfies (1). P. Soardi [17, Theorems I, II] proved that T has a fixed point if either X has normal structure [3] or T has diminishing orbital diameters [2]. In this paper, the following results are obtained: (a) T has a fixed point if it is continuous (with respect to the strong topology). In fact, T has a fixed point if it is continuous along line segments. It may be worthwhile to mention here that it is a well-known open problem that the same conclusion holds for nonexpansive self maps on X [1, p. 217]. (b) T has a fixed point if for any closed convex subset H of X with more than one point and $T(H) \subset H$,

$$\inf \{d(T(y), y): y \in H\} < \delta(H) .$$

It is obvious that this result generalizes the above results of Soardi and the above result of Kannan. (c) Let T be a self map on X such that there exist a_1, a_2, a_3, a_4, a_5 in $[0, 1]$ for which $a_1 + a_2 + a_3 + a_4 + a_5 = 1$ and for all x, y in X,