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REPRESENTATION OF SUPERHARMONIC FUNCTIONS MEAN CONTINUOUS AT THE BOUNDARY OF THE UNIT BALL

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In this paper it will be shown that superharmonic functions can be represented by a Green potential together with their boundary values if taken mean continuously at the boundary of the unit ball.

Introduction. It is well known that if $u(r, \theta, \phi)$ is harmonic inside the unit ball and has radial limit $\lim_{r\to 1} u(r, \theta, \phi) = 0$ everywhere on the surface, then u is not necessarily identically null inside and thus cannot be represented by its radial boundary values. Furthermore, there is an L_1 (Lebesgue class) harmonic function, see §2. *Remarks*, which satisfies $\lim_{r\to 1} u(r, \theta, \phi) = 0$ except for (1, 0, 0). In [1] and [3], Shapiro established the representation of harmonic functions in the two dimensional unit disc by their radial limits when a certain radial growth condition is satisfied. However, the set of functions satisfying the radial growth condition does not contain the class L_1, and conversely. Also, the analogues of [1] and [3] have not been established in the N-dimensional unit ball, $3 \leqq N$.

Our intention is to establish a representation of superharmonic functions in L_1 on the N-dimensional unit ball by their boundary values if taken mean continuously. Definitions and the statement of the theorems follow in the next section.

1. Preliminaries. We shall work in N-dimensional Euclidean space R^N, $3 \leqq N$, and shall use the following notation: $x = (x_1, \cdots, x_N)$ and $B(x, r)$ = the open N-ball centered at x with radius r; $\tilde{B}(x, r) = B(x, r) \cap B(0, 1)$; $|E|$, the Lebesgue measure of E; ∂E, the boundary of E; $\bar{\partial} B(x, r) = \partial B(0, 1) \cap B(x, r)$; $d\omega_N$, the natural surface area on $\partial B(0, 1)$; and subscripted A's, positive absolute constances though possibly different from one occurrence to another. For a point $y_0 \in \partial B(0, 1)$, $u(x)$ a measurable function on some $\tilde{B}(y_0, r_0)$, and $f(y)$ a function on $\partial B(0, 1)$, we set for $\rho \leqq r_0$

$$u_f(y_0, \rho) = |\tilde{B}(y_0, \rho)|^{-1} \int_{\tilde{B}(y_0, \rho)} |u(x) - f(y_0)|\, dx\ .$$

We use the notation $u(y_0, \rho)$ when $f \equiv 0$.

THEOREM 1. *Let $u(x)$ be superharmonic in $\Omega = B(0, 1)$. If*