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APPROXIMATION AND INTERPOLATION FOR SOME SPACES OF ANALYTIC FUNCTIONS IN THE UNIT DISC

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Let U be a bounded open subset of the complex plane C such that U and $C\setminus \bar{U}$ are connected. (If $B \subset C$, $\bar{B}$ denotes its closure in C.) $H^\infty(U)$ is the space of all bounded analytic functions defined on U. Let $S \subset U$ be the zero set of a nonzero function in $H^\infty(U)$.

Necessary and sufficient conditions on S are given for the existence of an open set $0 \supset \bar{U}\setminus(\bar{S}\setminus S)$ such that $H^\infty(0)$ and $H^\infty(U)$ have the same restrictions to S. If U is the unit disc $D = \{z : |z| < 1\}$ and S is as above, the following result holds for all the Hardy spaces $H^p(D)$, $0 < p \leq \infty$: Given $g \in H^p(D)$, there is a function f analytic in $C\setminus(\bar{S}\setminus S)$ such that $f|_D \in H^p(D)$ and $f = g$ on S.

If S and U are as above, $H^\infty(U)|_S$ denotes the set of restrictions $f|_S$ of all $f \in H^\infty(U)$. If $S = \{z_n\} \subset D$ satisfies $\Sigma_n(1 - |z_n|) < \infty$, Detraz [3] proved the following result

(*): *There exists an open set* $0 \supset \bar{D}\setminus(\bar{S}\setminus S)$ *such that*
$$H^\infty(0)|_S = H^\infty(D)|_S.$$

In this paper we give two extensions of this result. First we show that (*) holds for domains of a somewhat more general type than the unit disc D. Consider the following statement which is very similar to (*):

(**) There exists an open set V such that $\bar{V}\setminus(\bar{S}\setminus S) \subset D$ and
$$H^\infty(V)|_S = H^\infty(D)|_S.$$

It turns out that conditions (*) and (**) are equivalent, even with D replaced by a somewhat more general set.

We shall make some use of the theory of the classical H^p spaces. We refer to [4] or [9] in this connection. Before stating our first result, we mention some more notation. If f is a complex valued function defined for each $z \in B$ we put $||f||_B = \sup\{|f(z)|, z \in B\}$. If $U \subset C$ is open, $H^\infty(U)$ is a Banach algebra with sup norm on U and we denote by M the maximal ideal space of $H^\infty(U)$. The maximal ideals $m \in M$ are identified with the multiplicative functionals on $H^\infty(U)$ they correspond to. If $S \subset U$ is relatively closed and I denotes the set of all $f \in H^\infty(U)$ which are zero on