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BAER AND UT-MODULES OVER DOMAINS

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For a domain R, an R-module A is called a Baer module if $\operatorname{Ext}_R^1(A, T) = 0$ for every torsion R-module T. Dual to Baer modules, a torsion R-module B is called a UT-module if $\operatorname{Ext}_R^1(X, B) = 0$ for every torsion free R-module X. In this paper properties of these two types of modules will be derived and characterizations of Prüfer domains, Dedekind domains and fields will be obtained in terms of Baer and UT-module properties. One characterization will show the Baer modules are analogous to projective modules in the sense that a domain R is Dedekind if and only if, over R, submodules of Baer modules are Baer. In addition, just as a semisimple ring S can be characterized by the property that all S-modules are injective, or, equivalently, all S-modules are projective, a domain R is a field exactly if every torsion R-module is UT or, equivalently, every torsion free R-module is a Baer module. Further properties of these two kinds of modules will provide sufficient conditions to bound the global dimension of a domain R.

0. Historical Note. The concept of a Baer module goes back to 1936 when R. Baer, in [**1**], proposed the problem asking for a complete characterization of all abelian groups G such that $\operatorname{Ext}_Z(G, T) = 0$, for all torsion abelian groups T. At that time he showed that any such abelian group must be torsion free, and free if it had countable rank. Then in 1959 R. Nunke, in [**10**], extended these results to modules over a Dedekind domain, proving that such a module was again torsion free, and projective if it had countable rank. Finally in 1969 P. Griffith, in [**5**], completely solved Baer's problem for abelian groups, and showed that any such abelian group, now called a Baer group or B-group, must be free. In [**6**], the author extended Griffith's techniques to modules over a Dedekind domain, showing that if A is a Baer module over a Dedekind domain then A is projective. The major adjustments needed in this transition from abelian groups to modules over a Dedekind domain were accomplished by means of Corollary 2 on p. 279 of [**12**], Theorems 3 and 5 in [**8**], Lemma 8.3 and Theorem 8.4 in [**10**], as well as the exposition on ideals and valuations in §18, 19 of [**4**].

1. Preliminaries. Unless additional restrictions are stated, in this paper R denotes an arbitrary integral domain: that is, a commutative ring with 1 having no zero divisors. The quotient field of R will be denoted by Q.