PACIFIC JOURNAL OF MATHEMATICS
Vol. 54, No. 2, 1974

FIXED POINT THEOREMS FOR MULTIVALUED NONCOMPACT ACYCLIC MAPPINGS

P. M. FITZPATRICK AND W. V. PETRYSHYN

Let X be a Frechet space, D a closed convex subset of X, and $T: D \to 2^X$ an upper semicontinuous multivalued acyclic mapping. Using the Eilenberg-Montgomery Theorem and the earlier results of the authors, it is first shown that if $W \supset T(D)$ and $f: W \to D$ is a single-valued continuous mapping such that $fT: D \to 2^X$ is Φ-condensing, then fT has a fixed point. This result is then used to obtain various fixed point theorems for acyclic Φ-condensing mappings $T: D \to 2^X$ under the Leray-Schauder boundary conditions in case $D = \overline{\text{Int}(D)}$ and under the outward and /or inward type conditions in case $\text{Int}(D) = \phi$.

Introduction. Let X be a Frechet space and D an open or a closed convex subset of X. It is our object in this paper to establish fixed point theorems for not necessarily compact (e.g. condensing) multivalued acyclic mappings $T: D \to 2^X$ which need not satisfy the condition "$T(D) \subset D$" but instead are required to satisfy weaker conditions of the Leray-Schauder type. Our results are based upon the Eilenberg-Montgomery Theorem [**4**] and upon our Lemma 1 in [**16**]. The fixed point theorems presented in this paper for multivalued maps in infinite dimensional spaces strengthen and extend certain fixed point theorems of Górniewicz-Granas [7] and Powers [**17**] for acyclic compact maps, the results for star-shaped-valued maps of Halpern [**8**] for compact maps and our own [**16**] for condensing maps, and a number of fixed point theorems for convex-valued compact and noncompact maps (see Ky Fan [**5**], Browder [**1**], Reich [**18**], Ma [**12**], Walt [**20**], and [**20, 8, 15**] for related results and further references).

1. Let X be a Frechet space. If $D \subset X$, then we will denote by $\bar{D}$ and ∂D the closure and boundary of D, respectively.

DEFINITION 1. If C is a lattice with a minimal element, which we will denote by 0, then a mapping $\Phi: 2^X \to C$ is called a *measure of noncompactness* provided that the following conditions hold for any A, B in 2^X:

(1) $\Phi(A) = 0$ if and only if A is precompact.

(2) $\Phi(\overline{\text{co}}A) = \Phi(A)$, where $\overline{\text{co}}A$ denotes the convex closure of A.

(3) $\Phi(A \cup B) = \max\{\Phi(A), \Phi(B)\}$.