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HOMOMORPHISMS OF RIESZ SPACES

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If L is a Riesz space (lattice ordered vector space), a Riesz homomorphism of L is an order preserving linear map which preserves the finite operations "$\vee$" and "$\wedge$". It is shown here that if L is one of a large class of spaces and φ is a Riesz homomorphism from L onto an Archimedean Riesz space, then φ preserves the order limits of sequences.

The symbol θ will be used to denote the zero element of any vector space. Suppose L is a Riesz space (lattice ordered vector space). If $f \in L$ then $|f| = f \vee \theta - (f \wedge \theta)$. If M is a linear subspace of L then M is said to be an *ideal* of L if, whenever $|g| \leqq |f|$ and $f \in M$, then $g \in M$. If each of L_1 and L_2 is a Riesz space, a Riesz *homomorphism* φ from L_1 to L_2 is a linear map from L_1 to L_2 which preserves order and the finite operations "$\bigvee$" and "$\bigwedge$". A sequence $f_1, f_2, f_3, \cdots$ of points is said to *order converge* to the point f if there exists a sequence $u_1 \geqq u_2 \geqq u_3 \geqq \cdots$ and a sequence $v_1 \leqq v_2 \leqq v_3 \leqq \cdots$ of points such that $\bigvee v_p = f$, $\bigwedge u_p = f$, and $v_p \leqq f_p \leqq u_p$. Order convergence for nets is defined analogously. A sequence $f_1, f_2, f_3, \cdots$ of elements of the Riesz space L is said to converge *relatively uniformly* to the element f of L if there exists an element g of L (called the regulator) such that if $\varepsilon > 0$, there exists a number N_ε such that if n is a positive integer greater than N_ε, then $|f - f_n| \leqq \varepsilon g$. A Riesz space L is said to be *Archimedean* if, whenever f and g are two points of L such that $\theta \leqq nf \leqq g$ for all positive integers n, then $f = \theta$. Also L is said to be *σ-complete* if each countable set of positive elements has a greatest lower bound and *complete* if each set of positive elements has a greatest lower bound. If φ is a Riesz homomorphism which preserves the order limits of sequences then φ is said to be a Riesz *σ-homomorphism*. If φ preserves the order limits of nets it is said to be a *normal Riesz homomorphism*. A one-to-one onto map which is a Riesz homomorphism is a *Riesz isomorphism*. If H is a subset of L, H^+ will denote the set of all points f of H such that $f \geqq \theta$. If $f \in L$ then f^+ denotes $f \vee \theta$.

Suppose L is a Riesz space, M is an ideal of L, and the algebraic quotient L/M is partially ordered as follows: If each of H and K belongs to L/M and there is an element h of H and k of K such that $h \geqq k$, then $H \geqq K$. It follows that L/M is a Riesz space and the normal map $\pi\colon L \to L/M$ is a Riesz homomorphism (Luxemburg and Zaanen [3], p. 102). The coset of L/M containing f will be denoted $[f]$. Further, if M is the kernel of a Riesz homomorphism φ defined