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BEST APPROXIMATION BY A SATURATION CLASS OF POLYNOMIAL OPERATORS

D. S. GOEL, A. S. B. HOLLAND,
C. NASIM, AND B. N. SAHNEY

The problem of determining a saturation class has been considered by Zamanski, Sunouchi and Watari and others. Zamanski has considered the Cesaro means of order 1 and Sunouchi and Watari have studied the Riesz means of type n. The object of the present paper is to extend these results by considering Nörlund means which include the above-mentioned results as particular cases.

1. Let $\{p_n\}$ be a sequence of positive constants such that

$$P_n = p_0 + \cdots + p_n \longrightarrow \infty \quad \text{as} \quad n \longrightarrow \infty .$$

A given series $\sum_{n=0}^{\infty} d_n$ with the sequence of partial sums $\{S_n\}$ is said to summable (N, p_n) to d, provided that

$$(1.1)\qquad \begin{aligned} N_n\left[\sum_{l=0}^{\infty} d_l\right] &= \frac{1}{P_n}\sum_{k=0}^{n} P_{n-k} d_k \\ &= \frac{1}{P_n}\sum_{k=0}^{n} p_{n-k} S_k \longrightarrow d , \quad \text{as} \quad n \longrightarrow \infty , \end{aligned}$$

and N_n are called the Nörlund operators.

Let

$$(1.2)\qquad \frac{1}{2}a_0 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx) \equiv \sum_{k=0}^{\infty} A_k(x)$$

be the Fourier series associated with a continuous periodic function $f(x)$, with period 2π.

We define

$$(1.3)\qquad N_n(x) \equiv N_n(f; x) \equiv \frac{1}{P_n}\sum_{k=0}^{n} P_{n-k} A_k(x)$$

and the norm

$$||f(x) - N_n(x)|| \equiv \max_{0 \leq x \leq 2\pi} |f(x) - N_n(x)| .$$

If there exists positive nonincreasing function $\phi(n)$ and a class of functions K, with the following properties:

(I) $||f(x) - N_n(x)|| = o(\phi(n)) \Longrightarrow f(x)$ is constant,

(II) $||f(x) - N_n(x)|| = O(\phi(n)) \Longrightarrow f(x) \in K$