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CHARACTERS FULLY RAMIFIED OVER A NORMAL SUBGROUP

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Let H be a group and N a normal subgroup. Assume that χ is an irreducible (complex) character of H, and that the restriction of χ to N is a multiple of some irreducible character of N, say θ. Then $\chi_N = e\theta$, and e is called the ramification index. It is easy to see that it always satisfies $e^2 \leqq |H: N|$, and when equality holds, χ is said to be fully ramified over N. It is this "fully ramified case" which will be studied here in some detail. As an application of some of the methods of this paper, we prove the following solvability theorem in the last section. If H has an irreducible character fully ramified over a normal subgroup N and if p^4 is the highest power of p dividing $|H: N|$ for all primes corresponding to nonabelian Sylow p-subgroups of H/N, then H/N is solvable.

1. Fully ramified triples. Groups of type f.r. To simplify notation, say that (H, N, χ) is a fully ramified triple if χ is an irreducible character of H, N is normal in H, and χ is fully ramified over N. It has been conjectured in [13] that H/N is solvable in this case, and some partial results in this direction appear in [12]. We extend this work in Theorem 4.5 below. It is also possible to show that no known simple group can occur as a homomorphic image of H/N, but we will only need to consider a few cases in this paper (see Lemmas 4.1 and 4.3).

Since we are primarily concerned with the factor group H/N, rather than with H itself, the following theorem (due ultimately to I. Schur and A. H. Clifford) is extremely useful.

THEOREM 1.1. *Let H be a group, N a normal subgroup, and χ an irreducible character of H. Let θ be an irreducible constituent of χ_N, and assume $\chi_N = e\theta$* (i.e. *θ is invariant*). *Then, there exists a group H^*, with an irreducible character χ^*, and a normal subgroup N^* having a faithful irreducible character θ^*, such that*

$$\chi^*_{N^*} = e\theta^*$$
$$N^* \text{ is central in } H^*$$
$$\text{and } H/N \cong H^*/N^* .$$

Moreover, the isomorphism is "natural" in the sense that if K is any normal subgroup of H containing N, and K^*/N^* corresponds