PACIFIC JOURNAL OF MATHEMATICS
Vol. 55, No. 1, 1974

MEASURABLE UNIFORM SPACES

ZDENĚK FROLÍK

A uniform space is called $\aleph_0$-measurable if the pointwise limit of any sequence of uniformly continuous functions (real valued) is uniformly continuous. A uniform space is called measurable if the pointwise limit of any sequence of uniformly continuous mappings into any metric space is uniformly continuous.

It is shown that measurable spaces are just metric-fine spaces with the property that the cozero sets form a σ-algebra, or just hereditarily metric-fine spaces.

Metric-fine spaces seem to form a very useful class of spaces; they were introduced by Hager [5], and studied recently by Rice [7] and the author [2], [3]. Separable measurable spaces are studied in Hager [6].

The notation and terminology of Čech [1] is used throughout; for very special terms see Frolík [2]. The main result of the author's [3] is assumed, and [4] may help to understand the motivation.

If X is a uniform space we denote by $\operatorname{coz} X$, zX or BaX accordingly the cozero sets in X (i.e., the sets $\operatorname{coz} f = \{x \mid fx \neq 0\}$ where f is a uniformly continuous function), or the zero sets in X (i.e., the complements of the cozero sets), or the smallest σ-algebra which contains $\operatorname{coz} X$ (equivalently: zX). Since any uniform cover is realized by a mapping into a metric space, the completely coz-additive uniform covers form a basis for the uniformity. Completely coz-additive means that the union of each subfamily is a cozero set.

If X is a uniform space then eX is the set X endowed with the uniformity having the countable uniform covers of X for a basis of uniform covers; eX is a reflection of X in the class of separable uniform spaces (i.e., in spaces Y with $eY = Y$).

We denote by α the usual coreflection into fine uniform spaces. Recall that αX is the set X endowed with the finest uniformity which is topologically equivalent to the uniformity of X. The first theorem is a version of a simple classical result on measurable functions. The equivalence of Conditions 1–5 appears in Hager [6]. This theorem is repeatedly used in the sequel, and therefore an economical proof is furnished.

THEOREM 1. *Each of the following conditions is necessary and sufficient for a uniform space X to be $\aleph_0$-measurable.*

1. *eX is $\aleph_0$-measurable.*