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TWO RELATED INTEGRALS OVER SPACES OF CONTINUOUS FUNCTIONS

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In this paper the authors evaluate Yeh-Wiener integrals (which apply to functionals of a variable continuous function of two arguments) in terms of multiple Wiener integrals (which apply to functionals of several variable continuous functions of one argument). First somewhat specialized cases are given where the multiplicity of the Wiener integral is finite, and then quite general Yeh-Wiener integrals are evaluated in terms of limits of n-fold Wiener integrals as $n \to \infty$.

Introduction. James Yeh [5][1] defined Wiener measure in the space $C_2[S]$ of continuous real valued functions of two variables defined on the square $S: 0 \leqq s \leqq 1,\ 0 \leqq t \leqq 1$ and vanishing whenever s or t equals zero. More recently James Kuelbs [3, 4] extended Yeh's integral to integration over $C_2[X]$, the space of continuous real valued functions on any compact subset X of the plane. Kuelbs also defined a similar integral over spaces of functions of several variables and even infinitely many variables [4].

In the present paper we shall consider integration over $C_2[X]$ in the case where X is the rectangle $R = \{(s, t) \mid a \leqq s \leqq b, \alpha \leqq t \leqq \beta\}$. We note that this is closely connected with Yeh's integral over $C_2[S]$ and that Kuelbs has given a formula for relating integrals over $C_2[R]$ with integrals over $C_2[S]$, [3, p. 18].

Yeh's measure as applied to the space

$$C_2[R] \equiv \{x(\cdot, \cdot) \mid x(a, t) = x(s, \alpha) = 0,\ x(s, t) \text{ continuous for } a \leqq s \leqq b,\ \alpha \leqq t \leqq \beta\}$$

is defined as follows. Let $a = s_0 < s_1 < \cdots < s_m = b$, and $\alpha = t_0 < t_1 < \cdots < t_n = \beta$ be subdivisions of $[a, b]$ and $[\alpha, \beta]$ respectively and let $-\infty \leqq P_{j,k} \leqq Q_{j,k} \leqq +\infty$ be given for $j = 1, \cdots, m$ and $k = 1, \cdots, n$. Then

$$I = \{x \in C_2[R] \mid P_{j,k} < x(s_j, t_k) \leqq Q_{j,k} \text{ for } j = 1, \cdots, m,\ k = 1, \cdots, n\}$$

will be called an "interval" in $C_2[R]$. He defines the measure of the interval I by

[1] See also reference to Kitagawa in [5].