PACIFIC JOURNAL OF MATHEMATICS
Vol. 55, No. 2, 1974

REMARK ON MAPPINGS NOT RAISING DIMENSION OF CURVES

J. KRASINKIEWICZ

The purpose of this note is to prove three theorems on dimension raising ability of certain classes of maps defined on 1-dimensional continua. In particular we obtain a generalization of a recent result of J. Jobe concerning dimension raising ability of inverse arc functions defined on dendrites.

By a continuum we mean a compact connected metric space. A 1-dimensional continuum is called a curve. If each point of a continuum X has arbitrary small neighborhood with finite boundary, then X is said to be regular. X is suslinian provided any collection of mutually disjoint nondegenerate subcontinua of X is at most countable [6]. For a nondegenerate continuum we have the following implications:

(i) (regular) $\Rightarrow$ (suslinian) $\Rightarrow$ (1-dimensional).

Let f be a mapping of a continuum X into a continuum Y. We shall consider the following properties of f:

(α) for every arc $L \subset Y$ there exists an arc $M \subset X$ which is mapped by f onto L, i.e., $f(M) = L$.

(β) for every arc $L \subset Y$ there exists a continuum $M \subset X$ which is mapped by f onto L.

(γ) for every continuum $L \subset Y$ there exists a continuum $M \subset X$ which is mapped by f onto L.

THEOREM 1. *If f is a mapping with property (β) of a suslinian continuum X onto a locally connected continuum Y, then Y is suslinian.*

Proof. Suppose it is not true. Then there is an uncountable collection $\{B\}$ of nondegenerate mutually disjoint subcontinua of Y. Consider a member $B \in \{B\}$. Let a and b be distinct points of B. Let $U_1, U_2, \cdots$ be a decreasing sequence of neighborhoods of B (in Y) which limits on B, i.e.,

$$\bigcap_n U_n = B . \tag{1}$$

For each positive integer n there is a locally connected continuum C_n such that

$$B \subset C_n \subset U_n \qquad \text{(see [5], p. 260)} . \tag{2}$$