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METRIZABILITY OF TOPOLOGICAL SPACES

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This paper is a study of conditions under which a topological space is metrizable or has a countable base. In § 2 we consider the metrizability of spaces having a weak base in the sense of Arhangel'skiĭ. In § 3 we extend earlier work of Bennett on quasi-developments by showing that every regular θ-refinable β-space with a quasi-G_δ-diagonal is semi-stratifiable. One consequence of this result is a generalization of the Borges-Okuyama theorem on the metrizability of a paracompact $w\Delta$-space with a G_δ-diagonal. In § 4 we prove that a regular space has a countable base if it is hereditarily a CCC $w\Delta$-space with a point-countable separating open cover. This result is motivated by the remarkable theorem of Arhangel'skiĭ which states that a regular space has a countable base if it is hereditarily a Lindelöf p-space. In § 5 we show that every regular p-space which a Baire space has a dense subset which is a paracompact p-space. This result, related to work of Šapirovskiĭ, is then used to obtain conditions under which a Baire space satisfying the CCC is separable or has a countable base. In § 6 we prove that every locally connected, locally peripherally separable meta-Lindelöf Moore space is metrizable. Finally, in § 7 we consider the metrizability of spaces which are the union of countably many metrizable subsets. The results obtained in this section extend earlier work of Čoban, Corson-Michael, Smirnov, and Stone.

1. Preliminaries. We begin with some definitions and known results which are used throughout this paper. Unless otherwise stated, no separation axioms are assumed; however, regular, normal, and collectionwise normal spaces are always T_1 and paracompact spaces are always Hausdorff. The set of natural numbers is denoted by N, and $i, j, k, m, n, r, s,$ and t denote elements of N.

Let X be a set, let $\mathscr{G}$ be a collection of subsets of X, let p be an element of X. The *star of p with respect to $\mathscr{G}$*, denoted $\mathrm{st}(p, \mathscr{G})$, is the union of all elements of $\mathscr{G}$ containing p. The *order of p with respect to $\mathscr{G}$*, denoted $\mathrm{ord}(p, \mathscr{G})$, is the number of elements of $\mathscr{G}$ containing p. The union of all elements of $\mathscr{G}$ is denoted by $\mathscr{G}^*$. If $\mathscr{G}$ covers X, then $\mathscr{G}$ is said to be *separating* [43] if given any two distinct points p and q in X, there is some G in $\mathscr{G}$ such that $p \in G$, $q \notin G$.

A topological space X is said to be *developable* if there is a sequence $\mathscr{G}_1, \mathscr{G}_2, \cdots$ of open covers of X such that, for each p in X, $\{\mathrm{st}(p, \mathscr{G}_n): n = 1, 2, \cdots\}$ is a fundamental system of neighborhoods of