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THE LOCAL RIGIDITY OF THE MODULI SCHEME FOR CURVES

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Let Y be a smooth, quasi-projective scheme of finite type over an algebraically closed field of characteristic zero. Let X be the quotient of Y by a finite group of automorphisms. Assume that the branch locus of Y over X is of codimension at least 3. In this note, it is shown that X is locally rigid in the following sense: the singular locus of X is stratified and, given a point on a stratum, it is shown that there exists a locally algebraic transverse section to the stratum at the point which is rigid. This result is then applied to the coarse moduli scheme for curves of genus g, where $g > 4$ (in characteristic zero).

1. Stratifying quotient schemes. Let k be an algebraically closed field. Let V' be a smooth, irreducible quasi-projective algebraic k-scheme. By a *quotient scheme,* we mean a scheme $V = V'/G$, where G is a finite group of automorphisms of V'. In [3], Popp defines a stratification of such schemes.

Given a point $P \in V$ and a point $P' \in V'$ lying over P, one may define the inertia group of P':

$$I(P') = \{\sigma \in G \mid \sigma x \equiv x \bmod \mathcal{M}_{P'}, \quad \text{for all} \quad x \in \mathcal{O}_{V',P'}\} .$$

If $P'' \in V'$ is another point lying over P, then $I(P')$ and $I(P'')$ are conjugate subgroups of G.

Let Z_P denote the closed subscheme of Spec $(\mathcal{O}_P)$ which is ramified in the covering $f : V' \to V$ and let $Z_{P'}$ be the inverse image of Z_P in Spec $(\mathcal{O}_{P'})$. Denote by $Z'_1, \cdots, Z'_s$ those irreducible components of $Z_{P'}$ of dimension $n-1$ (where $n = \dim V$). Let $H_1, \cdots, H_s$ denote the inertia groups of the generic points of $Z'_1, \cdots, Z'_s$ respectively and let $H(P')$ denote the subgroup of $I(P')$ generated by the H_i, $i = 1, 2, \cdots, s$. (If $s = 0$, put $H(P') = (1)$.) Let

$$\bar{I}(P') = I(P')/H(P')$$

and call this the *small inertia group* of P'. Under the assumption that V' is smooth, Popp shows that $\bar{I}(P')$ is independent of the cover; i.e.,