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AN INTEGRAL REPRESENTATION FOR STRICTLY CONTINUOUS LINEAR OPERATORS

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Let B denote the algebra of bounded analytic functions on the open unit disc D in the complex plane. Let (B,τ) denote B endowed with the topology τ, where τ is chosen from κ, β or σ, respectively, the topology of uniform convergence on compact subsets of D, the strict topology and the topology of uniform convergence on D. This note obtains an integral representation of the form $Tf(z)=\int_\Gamma f(w)K(z,w)dw$ where $\Gamma=\{z:|z|=1\}$ for the linear operators which are continuous from (B,κ) into (B,σ). This representation is then used to study the convergence of operators in the full algebra of all continuous linear operators from (B,β) into (B,β).

1. Introduction. Let $M(D)$ denote the set of bounded complex valued Borel measures on D. R. C. Buck [5] showed that L is a continuous linear functional on $(C(D),\beta)$ if and only if $Lf=\int_D f d\mu$, $\forall f\in C(D)$ for some $\mu\in M(D)$. L. A. Rubel and A. L. Shields [7] showed that for any $\mu\in M(D)$ there exists a function h in $L^1(\Gamma)$ such that $\int_D f d\mu=\int_\Gamma f(x)h(x)dx$, $\forall f\in B$ and conversely, that any $h\in L^1(\Gamma)$ determines a measure $\mu\in M(D)$ for which this equality holds. Thus the continuous linear functionals on (B,β) can be represented as integration over Γ with respect to functions in $L^1(\Gamma)$.

Letting both τ_1 and τ_2 be one of the topologies κ,β or σ, let $[\tau_1:\tau_2]$ denote the algebra of all continuous linear operators from (B,τ_1) into (B,τ_2).

In Theorem 1 it is shown that any linear operator T in $[\beta:\beta]$ can be represented in the form

$$Tf(z)=\int_\Gamma f(w)K(z,w)dw, \qquad \forall f\in B.$$

However, a necessary and sufficient condition on $K(z,w)$ that such a T be in $[\beta:\beta]$ is not known.

The algebra $[\kappa:\sigma]$ is a dense subalgebra of $[\beta:\beta]$ in the compact open topology. In Theorem 3 it is shown that a linear operator T is in $[\kappa:\sigma]$ if and only if $Tf(z)=\int_\Gamma f(w)\ K(z,w)dw$ where the kernel