PACIFIC JOURNAL OF MATHEMATICS
Vol. 56, No. 1, 1975

ABELIAN GROUPS, A, SUCH THAT HOM$(A,\text{-})$ PRESERVES DIRECT SUMS OF COPIES OF A

D. M. ARNOLD AND C. E. MURLEY

An R-module, A, is *self-small* if Hom$(A,\text{-})$ preserves direct sums of copies of A. Various conditions on the endomorphism ring of a module which guarantee that it is self-small are studied. Various results are proved about subgroups of direct sums or direct products of copies of a self-small abelian group A, which generalize results previously known when A is torsion free of rank one.

0. Introduction. An R-module, A, is *self-small* if $\mathrm{Hom}_R(A,\text{-})$ preserves direct sums of copies of A. Homological arguments show that if A is a self-small R-module with R a commutative ring with 1, then the category of direct summands of direct sums of copies of A is equivalent to the category of projective right $\mathrm{End}_R(A)$-modules ($\mathrm{End}_R(A)$ is the R-endomorphism ring of A). Consequently, direct sum decompositions of direct sums of copies of A may be interpreted in terms of direct sum decompositions of free $\mathrm{End}_R(A)$-modules.

An R-module, A, is self-small in the following cases: (a) A is *small* (i.e., $\mathrm{Hom}_R(A,\text{-})$ preserves arbitrary direct sums of R-modules); (b) $A = \Pi_{i\in I} A_i$, where each A_i is a self-small R-module and $\mathrm{Hom}_R(A_i, A_j) = 0$ if $i \neq j$; (c) $\mathrm{End}_R(A)$ is countable.

If the finite topology on $\mathrm{End}_R(A)$ is discrete, then A is self-small. In certain cases, the converse is true.

COROLLARY I. *Suppose that A is a countably generated R-module. Then A is self-small iff the finite topology on $\mathrm{End}_R(A)$ is discrete. If R is countable, then A is self-small iff* $\mathrm{End}_R(A)$ *is countable.*

A left ideal, I, of $\mathrm{End}_R(A)$ is an *annihilator ideal* if $I = \{f \in \mathrm{End}_R(A)\colon f(x) = 0 \text{ for all } x \in A \text{ with } Ix = 0\}$.

PROPOSITION II. *Suppose that A is an R-module and that* $\mathrm{End}_R(A)$ *has the minimum condition on left annihilator ideals. Then the finite topology on* $\mathrm{End}_R(A)$ *is discrete and A is the finite direct sum of indecomposable R-modules.*

The remainder of the paper is devoted to self-small abelian groups (although many of the arguments are valid in a more general setting). Self-small torsion abelian groups are finite. Section 3 and examples in §5 demonstrate that self-small torsion free abelian groups are both profuse and diverse. Self-small mixed abelian groups with finite torsion free rank are characterized by Proposition 3.6.