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SEMIPRIME RINGS WITH THE SINGULAR SPLITTING PROPERTY

MARK L. TEPLY

A (right nonsingular) ring R is called a splitting ring if, for every right R-module M, the singular submodule $Z(M)$ is a direct summand of M. If R is a semiprime splitting ring with zero right socle, then R contains no infinite direct sum of two-sided ideals. As applications of this result, the center of a semiprime splitting ring with zero socle is analyzed, and the study of splitting ring is completely reduced to the case where R is a prime ring. The center of a semiprime splitting ring is a von Neumann regular ring.

1. Introduction. In this paper R denotes an associative ring with identity element. Unless otherwise noted, all modules will be unital right modules.

Considerable work has recently been done on the problem of characterizing the rings for which the singular submodule $Z(M)$ of every module M is a direct summand. Such rings will be called *splitting rings* in this paper. Every splitting ring is a right nonsingular ring. Rotman [6] showed that a commutative integral domain is a splitting ring if and only if it is a field. Cateforis and Sandomierski [1] characterized the commutative splitting rings as the von Neumann regular rings R with the property that, for every essential ideal I of R, R/I is a direct sum of fields. In a series of papers [2, 3, 4], Goodearl (a) reduced the study of splitting rings to the study of rings with essential right socle or zero right socle, (b) characterized the splitting rings with essential right socle, and (c) showed via a triangular matrix ring structure theorem that, in order to complete the characterization of splitting rings, it is sufficient to study the semiprime splitting rings with zero right socle.

In Theorem 7 of this paper, we show that a semiprime splitting ring with zero right socle is an essential product of *finitely* many prime splitting rings with zero right socle. (A ring R is an essential product of the rings $R_1, R_2, \cdots, R_n$, if R is a subdirect product of $R_1, R_2, \cdots R_n$ which contains an essential right ideal of $\prod_{i=1}^{n} R_i$.) Each prime ring used for the essential product in Theorem 7 is a homomorphic image of R which is determined in a natural way; so the product of prime rings is constructable from R. Moreover, *Theorem 7 can be used in the following way to reduce the study of splitting rings to the case where R is a prime ring with zero right socle.* By the discussion above, we only need to construct the semiprime