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# HOMOTOPY INVARIANCE OF CONTRAVARIANT FUNCTORS ACTING ON SMOOTH MANIFOLDS

BRIAN K. SCHMIDT

**It is sometimes possible to prove that a functor is homotopy invariant using only a knowledge of the domain and range categories of the functor. It is known, for example, that every covariant or contravariant functor from the category of simplicial complexes (with continuous mappings) to the category of countable groups is homotopy invariant. This result has been extended to covariant, but not contravariant, functors with domain the category of smooth manifolds. In the contravariant case, the proof breaks down because certain mappings are not differentiable. This fault will be corrected in this paper. Among other results, it will be shown that every contravariant functor from the category of smooth manifolds to the category of countable groups is homotopy invariant.**

The results mentioned above are proved in [4]. As in [4], we will use the word "cofunctor" to mean a contravariant functor. $\mathscr{C}$ will denote any full subcategory of the category of smooth manifolds which contains the real line $\boldsymbol{R}$ and is closed under the operation product-with-$\boldsymbol{R}$. $\mathscr{G}$ will denote any subcategory of the category of sets in which every object is countable. Let $C^\infty(\boldsymbol{R}, \boldsymbol{R})$ denote the monoid of smooth mappings from $\boldsymbol{R}$ to $\boldsymbol{R}$ under composition. Let $D$ denote the monoid dual to $C^\infty(\boldsymbol{R}, \boldsymbol{R})$. In light of paragraphs 15 and 16 of [4], Theorem 11 of [4] may be restated as follows:

THEOREM 2. *If $D$ cannot act faithfully on any countable set, then every cofunctor $\Delta: \mathscr{C} \to \mathscr{G}$ is homotopy invariant.*

The revised approach.

3. Suppose that $D$ acts faithfully on a set $B$. We will prove that $B$ is uncountable. Let $I$ denote the closed interval $[0, 1]$. For each $x \in I$, let $P_x$ be the set of all $p \in D$ such that the following two conditions are satisfied:

4. $w \in (x, 1) \Rightarrow p(w) \in (x, 1)$
5. $w \notin (x, 1) \Rightarrow p(w) = w$.

It is easy to verify that:

6. If $p' \in P_w$, $p \in P_x$, and $w \leqq x$, then $pp' \in P_w$.
7. Observe that every subset of $I$ has a greatest lower bound in $I$. Hence we may define, for each $b \in B$, a number $\lambda(b) \in I$ which is