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THE FIXED POINT PROPERTY FOR TREE-LIKE CONTINUA WITH FINITELY MANY ARC COMPONENTS

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It is shown that if M is a tree-like continuum with a finite number of arc components, then every continuous mapping of M into itself has a fixed point.

A continuum M is a compact, connected metric space. A continuum is said to be tree-like if for every $\varepsilon > 0$, there is an ε-cover of M whose nerve is a simple tree (a connected, one-dimensional, acyclic simplicial complex). In [1] Bing raised the question of whether these continua have the fixed point property. This is one of the most famous unsolved fixed point questions for continua. This paper provides an affirmative answer to Bing's question in the case where the tree-like continuum M has finitely many arc components.

Since tree-like continua are hereditarily unicoherent, it is easily seen that any subcontinuum of a tree-like continuum with finitely many arc components has finitely many arc components (see the proof of Lemma 1.3 below). It follows that any such continuum is hereditarily decomposable (indecomposable continua have uncountably many composants). Continua which are hereditarily decomposable and hereditarily unicoherent are called λ-dendroids. These continua were shown by Cook to be tree-like in [3].

The theorem presented here is thus a special case of the fixed point question for λ-dendroids which was raised by Knaster in [5][1]. Numerous special cases of this question have already been answered. For a survey of these results see [8], Chapter II. In particular Hamilton [4] has shown that all λ-dendroids have the fixed point property for homeomorphisms and Borsuk [2] has shown that λ-dendroids which are arcwise connected (dendroids) have the fixed point property for all continuous maps. The theorem presented here generalizes the latter result.

The paper is in two sections. The first section deals with density properties of arc components in λ-dendroids. Not all of the results in §1 are required in §2, which contains the proof of the fixed point theorem. The other material in §1 is included because the authors feel that it has some independent interest and because it raises a

[1] During revision of this paper for publication in this Journal, the authors received a manuscript from Roman Manka containing a theorem which implies that λ-dendroids have the fixed point property.