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ALGEBRAIC MAXIMAL SEMILATTICES

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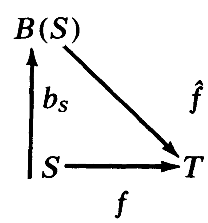
A topological semigroup S is maximal if it is closed in each topological semigroup that contains it. The semigroup S is called absolutely maximal if each continuous image is maximal. In this paper we are concerned with those discrete semilattices that are absolutely maximal. Thus we are concerned with those algebraic conditions on a semilattice which force it to be topologically closed.

In [9] Stralka studies those semigroups which have the congruence extension property. The semilattices we are concerned with and all their homomorphic images have this property. In fact, every congruence on such a semilattice S is closed. Thus S admits a compact Hausdorff topology $\mathcal{F}(S)$ under which multiplication is continuous. By [5] S admits a unique such topology. Also, since S has the congruence extension property for finite subsemilattices, the topology $\mathcal{F}(S)$ has a base which consists of subsemilattices [3].

In §II we give definitions, and we give necessary and sufficient conditions for a sublattice of a compact lattice to be closed. In §III we characterize those discrete semilattices and lattices which are absolutely maximal. Also, we show $(S, \mathcal{F}(S))$ is stable and 0-dimensional. In §IV we indicate how absolutely maximal discrete semilattices are constructed from a class of simple examples.

II. Definitions. Let S denote a topological semilattice. The Bohr compactification of S is a pair $(B(S), b_s)$ where $B(S)$ is a compact semilattice, $b_s\colon S \to B(S)$ is a continuous homomorphism and if $f\colon S \to T$ is a continuous homomorphism with T a compact semilattice, then there is a unique continuous homomorphism which makes the following diagram commute:



For the existence of the Bohr Compactification see either [1] or [2].