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COVERING THEOREMS FOR FINITE NONABELIAN SIMPLE GROUPS. V.

J. L. BRENNER, R. M. CRANWELL, AND J. RIDDELL

In the alternating group A_n, $n = 4k + 1 > 5$, the class C of the cycle $(12 \cdots n)$ has the property that CC covers the group. For $n = 16k$ there is a class C of period $n/4$ in A_n such that CC covers A_n; C is the class of type $(4k)^4$.

1. Introduction. It was shown by E. Bertram [1] that for $n \geqq 5$ every permutation in A_n is the product of two l-cycles, for any l satisfying $[3n/4] \leqq l \leqq n$. Hence A_n can be covered by products of two n-cycles and also by products of two $(n-1)$-cycles. But if n is odd the n-cycles in A_n fall into two conjugate classes C, C', and similarly for the $(n-1)$-cycles if n is even, so that the quoted result does not decide whether

$$CC = A_n. \tag{1}$$

The question was decided affirmatively for $n = 4k + 2$ and negatively for $n = 4k$, $4k - 1$ in [2]. The question is now decided affirmatively in the remaining case $n = 4k + 1$, $n \neq 5$.

THEOREM 1. *For $n = 4k + 1 > 5$, the class C of the cycle $(12 \cdots n)$ has property* (1).

The proof is in §§2–4.

Regarding the product CC', it was shown in [2] that CC' covers A_n $(n \geqq 5)$ if $n = 4k$, $4k - 1$, while if $n = 4k + 1$, $4k + 2$, CC' contains all of A_n but the identity.

By an argument quite similar to the proof of Theorem 1, we have proved

THEOREM 2. *For $n = 16k$, the class C of type $(4k)^4$ in A_n has property* (1).

The proof and some related matters are discussed in §5. Note that the class in Theorem 2 has period $n/4$.

2. The case $n = 9$. Let $a = (123456789)$. For every class in A_9, a conjugate b of a can be found such that ab represents (lies in) that class. This assertion is the substance of the table below.