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MAPPINGS BETWEEN ANRs THAT ARE FINE HOMOTOPY EQUIVALENCES

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It is shown in this note that every closed UV^∞ - map between separable ANRs is a fine homotopy equivalence.

We extend Lacher's result **[6,7]** that a closed UV^∞-map between locally compact, finite dimensional ANRs is a fine homotopy equivalence to the case of arbitrary separable ANRs. It is hoped that this theorem will be useful in studying manifolds modelled on the Hilbert Cube. (See **[1]**, section PF3. Added in proof. See also **[9]**).

A set $A \subset X$ has property UV^∞ if for each open set U of X containing A, there is an open V, with $A \subset V \subset U$ such that V is null-homotopic in U. A mapping $f: X \to Y$ of X onto Y is a UV^∞-map if for each $y \in Y$, $f^{-1}(y)$ is a UV^∞ subset of X. The mapping f is said to be closed if the image of every closed set is closed and proper if the inverse image of every compact set is compact. An absolute neighborhood retract for metric spaces is denoted an ANR. If α is a cover of Y and g_1 and g_2 are maps of a space A into Y, g_1 is α-near g_2 if for each $a \in A$ there is a $U \in \alpha$ containing $g_1(a)$ and $g_2(a)$. The map g_1 is α-homotopic to $g_2, g_1 \overset{\alpha}{\simeq} g_2$, if there is a homotopy $\lambda: A \times I \to Y$ taking g_1 to g_2 with the property that for each $a \in A$ there exists $U \in \alpha$ containing $\lambda(\{a\} \times I)$. A map $f: X \to Y$ is a fine homotopy equivalence if for each open cover, α, of Y there exists a map $g: Y \to X$ such that $fg \overset{\alpha}{\simeq} id_Y$ and $gf \overset{f^{-1}(\alpha)}{\simeq} id_X$.

Various versions of Lemma 3 have been proven by Smale **[8]**, Armentrout and Price **[2]**, Kozlowski **[5]** and Lacher **[6]**. The difference in this lemma is that K is not required to be a finite dimensional complex.

Let K be a locally finite complex and j be a nonnegative integer. When there is no confusion we will not distinghish between the complex K and its underlying point set $|K|$. If σ is a simplex of K, then $N(\sigma, K) = \{\tau < K \mid \sigma \cap \tau \neq \phi\}$ and $\mathrm{st}(\sigma, K) = \{\tau < K \mid \sigma < \tau\}$. Also K^j will denote the j-skeleton of K and ${}^jK = \{\sigma < K \mid |N(\sigma, K)| \subset |K^j|\}$. Let $\mathcal{U}$ be a covering of a space Y and B a subset of Y. The star of B with respect to $\mathcal{U}$, $\mathrm{st}^1(B, \mathcal{U})$, is the set $\{U \in \mathcal{U} \mid B \cap U \neq \phi\}$. Inductively, $\mathrm{st}^n(B, \mathcal{U})$ is defined to be $\mathrm{st}(\mathrm{st}^{n-1}(B, \mathcal{U}))$. A covering $\mathcal{V}$ is called a star^n refinement of $\mathcal{U}$ if the covering $\{\mathrm{st}^n(V, \mathcal{V}) / V \in V\}$ refines $\mathcal{U}$. Every open covering of a