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RELATIVELY INVARIANT MEASURES

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A homomorphism of minimal flows $X \xrightarrow{\phi} Y$, has a relatively invariant measure if there exists a positive projection from $\mathscr{C}(X)$ onto $\mathscr{C}(Y)$ which commutes with translasion. Such a relatively invariant measure does not always exists. However, some elementary facts from the theory of compact convex sub-sets of a locally convex topological vector space are used to show that given a homomorphism of minimal flows $X \xrightarrow{\phi} Y$ there exists a commutative diagram

$$\begin{array}{ccc} X^{\sim} & \xrightarrow{\theta^{\sim}} & X \\ {\scriptstyle \phi^{\sim}}\downarrow & & \downarrow{\scriptstyle \phi} \\ Y^{\sim} & \xrightarrow[\theta]{} & Y \end{array}$$

where θ and $\theta^{\sim}$ are strongly proximal homomorphisms and $\phi^{\sim}$ has a relatively invariant measure, (RIM). Homomorphisms which have invariant measures are studied and questions of existence and uniqueness are investigated.

Similar diagrams, where θ and $\theta^{\sim}$ are replaced by other types of proximal extensions, and $\phi^{\sim}$ is replaced by an open map with certain additional properties, are studied in [12] and [2].

In section one we introduce notions and definitions. Section two is devoted to the proof of the main theorem about affine flows and then some corollaries for homomorphisms of minimal flows are deduced. Another corollary is a generalization of the Ryll Nardzewskie fixed point theorem. This results are extensions of results in [6].

In section three the notation of a relatively invariant measure is discussed and it is shown that metric distal extension has a relatively invariant measure (see [8] and [1]). A homomorphism with a RIM which has at least one finite fiber is shown to be almost periodic. In section four we show the existence of the commutative diagram mentioned above. This is used to show the existence of a universal strongly proximal extension for any given minimal flow. We conclude with some questions about the uniqueness of a RIM, and the existence of almost periodic extensions.