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EXTENSION FUNCTIONS FOR RANK 2, TORSION FREE ABELIAN GROUPS

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The set of isomorphism classes of rank 2, torsion free abelian groups with a pure subgroup isomorphic to a given rank 1 group is shown to be in natural 1−1 correspondence with the set of pairs consisting of a quotient type and a type of an extension function. In terms of these invariants, necessary and sufficient conditions are determined for such a group to be homogeneous or to admit a pure cyclic subgroup. Moreover, this 1−1 correspondence has an explicit inverse, so that examples are readily obtained.

Our method is to combine the Korosh-Malcev-Derry matrix classification (see **[2]** for this and other well known aspects of the theory of abelian groups which we employ) with some elementary observations converning abelian extensions of rank 1, torsion free abelian groups. We begin §1 by identifying $\mathrm{Ext}_Z(X, Y)$ for rank 1, torsion free abelian groups X and Y in terms of "extension functions." A Korosh-Malcev-Derry matrix sequence for the total group of such an extension is readily given in terms of an extension function. Moreover, an extension function explicitly determines a subgroup of $Q \oplus Q$. We obtain an explicit necessary condition for two extension functions to determine isomorphic total groups, as well as express the Korosh-Malcev-Derry matrix conditions in terms of extension functions.

In §2, we explicate the $1-1$ correspondence asserted above. We then "list" all homogeneous, rank 2, torsion free abelian groups of a given type. We also determine in terms of our invariants whether or not a rank 2, torsion free abelian group admits a pure cyclic subgroup.

We gratefully acknowledge many helpful conversations with C. Miller. Moreover, the referee's suggestion of generalizing an eariler version of this work proved most valuable. We refer the interested reader to (**[1]**) for a detailed study of rank 2, torsion free abelian groups up to quasi-isomorphism and to (**[2]**) for a survey of the literature on torsion free abelian groups.

1. Extension functions. A sequence $(a_1, \cdots, a_n, \cdots)$ with each a_i an extended nonnegative integer, $0 \leqq a_i \leqq \infty$, is called a characteristic. Two characteristics, $(a_1, \cdots, a_n, \cdots)$ and $(b_1, \cdots, b_n, \cdots)$, are said to have the same type if and only if $\Sigma_i (a_i - b_i)^2 < \infty$. If A is a torsion free abelian group, then the characteristic of any nonzero element x in A, $\mathrm{char}(x) = (a_1, \cdots, a_n, \cdots)$, is defined by