PACIFIC JOURNAL OF MATHEMATICS
Vol. 59, No. 1, 1975

GENERALIZED AXISYMMETRIC ELLIPTIC FUNCTIONS

PETER A. MCCOY

A generalized axisymmetric elliptic function (GASE) $\Psi_\nu: \Omega \subset E^n \to C$ of order $\nu \geqq 0$ solving the partial differential equation

$$(1) \quad \mathscr{L}_\nu(\Psi_\nu) \equiv \frac{\partial^2 \Psi_\nu}{\partial x^2} + \frac{\partial^2 \Psi_\nu}{\partial \rho^2} + \frac{2\nu}{\rho}\frac{\partial \Psi_\nu}{\partial \rho} + a(x)\frac{\partial \Psi_\nu}{\partial x} + c(x)\Psi_\nu = 0$$

with analytic coefficients is subject to Cauchy data: $\Psi_\nu(x, 0) = f(x)$, $(\partial/\partial\rho)(\Psi_\nu(x, 0)) = 0$ along the singular line. These GASE may be generated from associated analytic functions of one complex variable or associated solutions to the corresponding nonsingular equation by certain integral operators. Convexity arguments geometrically characterize the values of GASE from those of the associates and kernel functions of the respective operators.

An extensive theory based on integral operators which characterizes the distribution of singularities of various classes of GASE from the distribution of singularities of their associates was developed by S. Bergman [2], R. P. Gilbert [3, 4], P. Henrici [6] and their colleagues [5]. Our aim is to apply convexity arguments from the analytic theory of polynomials of one complex variable to develop a geometric theory of the value distribution of GASE from the known value distribution of the associates. These results are based on two operators developed by Henrici [3, p. 199]; one which utilizes a kernel function to generate GASE from associated analytic functions of one complex variable and one which generates GASE of positive order from the associated GASE of order zero.

A theory connecting the values of axisymmetric harmonic polynomials (AHP) in E^n with those of associated polynomials of one complex variable was developed by M. Marden [8]. Gegenbauer's integral for ultraspherical polynomials was used to map polynomials of one complex variable onto AHP and then convexity arguments were used to relate their values. Using geometrical methods and R. P. Gilbert's operater A_μ [3, p. 168], the auther [10-12] utilized the conformal mapping properties of the associates to characterize sets of excluded values for generalized axisymmetric potentials (GASP) corresponding to solutions of (1) with $a(x) \equiv c(x) \equiv 0$.

Convexity arguments used in studying GASP were essentially independent of the kernel of the operator A_μ which is non-negative and dependent only on the variable of integration. In general, oper-