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ADJUNCTIONS AND COMONADS IN DIFFERENTIAL ALGEBRA

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It is known that the construction of the ring of fractions $S^{-1}A$ of a commutative ring A by a multiplicative subset S of A can be extended to the differential case. This means that for a given differential ring (A, d), the differential ring of fractions of (A, d) by S is constructed simply by defining a derivation operator on $S^{-1}A$ in terms of the derivation operator d on A. We seek to explain in the categorical setting of adjunctions and comonads the reasons for which this and other constructions can be extended to the differential case. A natural product of this investigation is the construction of the differential affine scheme of a differential ring.

1. Introduction. Stated simply, there are three points which explain why certain constructions involving commutative rings can be carried over to the differential case. These three points are adjunction, comonad and compatibility. The reader is referred to [9] for the necessary background on adjunctions and monads (to which comonads are dual). We add a few words to clarify each of these points.

By adjunction we mean that each of the constructions we consider is part of an adjunction, i.e., is an adjoint functor. This point will be made clearer as we discuss each example in §§ 3, 4 and 5.

By comonad we mean that for each of the categories related to commutative rings there is a comonad on that category whose coalgebras are isomorphic to the differential analogue of that category. For example, the category **Diff** of differential rings is isomorphic to the category $\mathbf{Comm}_\Omega$ of Ω-coalgebras for a comonad Ω on the category **Comm** of commutative rings [7]. Since this example is of central importance for this paper, and since each of the other comonads we shall discuss is defined in terms of Ω, we elaborate on this point below.

For the remainder of this paper we adopt the convention that all rings are commutative with unit and all ring homomorphisms preserve the unit. We also make frequent use of the notation F: $\mathscr{A} \to \mathscr{B}$: $A \to FA$: $f \to Ff$ when defining a functor $F: \mathscr{A} \to \mathscr{B}$ to describe its action upon objects $A \in \mathscr{A}$ and morphisms $f \in \mathscr{A}$.

The category **Diff** has as its objects differential rings which are pairs (A, d) where A is a ring and d is a derivation operator on A, i.e., $d: A \to A$ is additive and satisfies the product rule $d(ab) =$