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ON CONJUGATE BANACH SPACES WITH THE RADON-NIKODÝM PROPERTY

TSANG-HAI KUO

It is shown that if the unit ball $B_{X^{**}}$ of X^{**} is Eberlein compact in the weak* topology, or if X^* is isomorphic to a subspace of a weakly compactly generated Banach space then X^* possesses the Radon-Nikodým property (RNP). This extends the classical theorem of N. Dunford and B. J. Pettis. If X is a Banach space with X^{**}/X separable then both X^* and X^{**} (and hence X) have the RNP. It is also shown that if a conjugate space X^* possesses the RNP and X is weak* sequentially dense in X^{**} then $B_{X^{**}}$ is weak* sequentially compact. Thus, in particular, if X^{**}/X is separable then $B_{X^{***}}$ is weak* sequentially compact.

1. Introduction. A Banach space X is said to have the *Radon-Nikodým property* (RNP) if for each positive finite measure space $(\Omega, \Sigma, \lambda)$ and every λ-continuous vector measure $\mu: \Sigma \rightarrow X$ with finite variation, there exists a Bochner integrable function $f: \Omega \rightarrow X$ such that

$$\mu(A) = Bochner \int_A f(\omega) d\lambda \ for\ all\ A \in \Sigma$$

The classical theorems of Dunford and Pettis [3] and Phillips [6] show that every separable conjugate space and every reflexive Banach space has RNP.

Recent work aimed at extending the Radon-Nikodým theorem to vector measures has yielded more general theorems which characterizes Banach spaces with the Radon-Nikodým property. For the purposes of this paper, we only list those that will be employed and refer to [8] for a more detailed introduction.

The two following theorems are essentially due to Uhl [9].

THEOREM 1.1. *Let X be a Banach space. Then the following statements are equivalent*:

(i) *X possesses RNP*;

(ii) *every subspace (by a subspace, we refer to a closed infinite—dimensional linear submanifold) of X possesses* RNP;

(iii) *every separable subspace of X possesses* RNP.

For a Banach space X, denote by X^* its conjugate space.