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INFIMUM AND DOMINATION PRINCIPLES IN VECTOR LATTICES

PETER A. FOWLER

The main purpose of this paper is to demonstrate (a) that the potential theoretic notions of infimum principle and domination principle are meaningful in a setting of a vector lattice with a monotone map to the dual space, and (b) in this general setting these two principles are equivalent under very weak hypotheses.

THEOREM 1. **If $T: L \to L'$ is a strictly monotone map, L a vector lattice, then the infimum principle implies the domination principle.**

THEOREM 2. **If $T: B \to B'$ is a monotone, coercive hemicontinuous map, B a reflexive Banach space and vector lattice with closed positive cone, then the domination principle implies the infimum principle.**

1. Introduction. The results herein improve theorems proved earlier by the author [7], and are motivated by work of Calvert [4] and Kenmochi and Mizuta [11, 12]. The paper [4] deals with the Sobolev space $W^{1,m}$ and a monotone operator satisfying certain further conditions. The papers [11, 12] deal with functional spaces whose intersection with either $\mathscr{C}$ (continuous compact support functions) or L^2 is a dense subspace and with a monotone operator which is the gradient of a certain convex function.

The present paper shows that the relationship between the infimum and domination principles is independent of the specialized properties of the above mentioned spaces. This relationship depends more on the lattice structure in a linear space in one case and on the lattice structure in a reflexive Banach space in the other case. Results on monotone operators of Browder [2, Theorem 1] and Hartman and Stampacchia [10, Theorem 1.1] are employed in the Banach space case.

2. Difinitions. Let L be a vector lattice with partial order $\ll$, and $L^+ = \{x \in L \mid 0 \ll x\}$. The symbol $x \wedge y$ denotes the infimum of x and y. Let L' be a subspace (possibly improper) of the algebraic dual of L. The usual bi-linear form on $L \times L'$ is denoted $\langle \cdot, \cdot \rangle$, and $L'^+ = \{f \in L' \mid \langle x, f \rangle \geqq 0 \text{ for all } x \in L^+\}$.

DEFINITION 1. A map $T: L \to L'$ is *monotone* if for all $x, y \in L$, $\langle x - y, Tx - Ty \rangle \geqq 0$; T is *strictly monotone* if T is monotone and $\langle x - y, Tx - Ty \rangle = 0$ implies $x = y$.