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θ-CLOSED SUBSETS OF HAUSDORFF SPACES

R. F. DICKMAN, JR. AND JACK R. PORTER

A topological property of subspaces of a Hausdorff space, called θ-closed, is introduced and used to prove and interrelate a number of different results. A compact subspace of a Hausdorff space is θ-closed, and a θ-closed subspace of a Hausdorff space is closed. A Hausdorff space X with property that every continuous function from X into a Hausdorff space is closed is shown to have the property that every θ-continuous function from X into a Hausdorff space is closed. Those Hausdorff spaces in which the Fomin H-closed extension operator commutes with the projective cover (absolute) operator are characterized. An H-closed space is shown not to be the countable union of θ-closed nowhere dense subspaces. Also, an equivalent form of Martin's Axiom in terms of the class of H-closed spaces with the countable chain condition is given.

1. Preliminaries. For a space X and $A \subseteq X$, the *θ-closure* of A, denoted as $\mathrm{cl}_\theta A$, is $\{x \in X$: every closed neighborhood of x meets $A\}$. The subset A is θ-closed if $\mathrm{cl}_\theta A = A$. Similarly, the *θ-interior* of A, denoted as $\mathrm{int}_\theta A$, is $\{x \in X$: some closed neighborhood of x is contained in $A\}$. Clearly, $\mathrm{cl}_\theta A$ is closed and $\mathrm{int}_\theta A$ is open. The concept of θ-closure was introduced by Velicko [15] and used by the authors in [3]. Also introduced in [15] is the concept of a H-set: a subset A of a Hausdorff space X is an *H-set* if every cover of A by sets open in X has a finite subfamily whose closures in X cover A; this concept was independently introduced in [11] and called *H-closed relative to X*. An *open filter* is a filter with a filter base consisting of open sets. A maximal open filter is called an *open ultrafilter*. A filter $\mathscr{F}$ on X is said to be *free* if $\mathrm{ad}_X \mathscr{F} \neq \emptyset$, otherwise, $\mathscr{F}$ is said to be *fixed*. A subset A of X is *far from the remainder* (f.f.r.) [1] in X if for every free open ultrafilter $\mathscr{U}$ on X, there is open $U \in \mathscr{U}$ such that $\mathrm{cl}_X U \cap A = \emptyset$; a subset A of X is *rigid in* X [3] if for every filter base $\mathscr{F}$ on X such that $A \cap \bigcap \{\mathrm{cl}_\theta F : F \in \mathscr{F}\} = \emptyset$, there is open set U containing A and $F \in \mathscr{F}$ such that $\mathrm{cl}\, U \cap F = \emptyset$. The following facts are used in the sequel:

(1.1) In $A \subseteq B \subseteq X$ and A is θ-closed in X, then A is θ-closed in B.

(1.2) A compact subset of a Hausdorff space is θ-closed.

(1.3) [15] *A* θ-closed subset of an H-closed space is an H-set.

(1.4) [3] Let A be a subset of a space X. The following are