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ALMOST PERIODIC HOMEOMORPHISMS OF E^2 ARE PERIODIC

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In this paper we show that every almost periodic homeomorphism of the plane onto itself must be periodic. This improves well-known results.

1. Introduction. In [3] Foland showed that every almost periodic homeomorphism of a *disk* onto itself is topologically either a reflection in a diameter or a rotation. Hemmingsen [7] studies homeomorphisms on compact subsets of E^2, with equicontinuous families of iterates, and shows that if such a compact set has an interior point of infinite order, then the compact set is a disk or annulus. If it is a disk, then the homeomorphism is a rotation or reflection. Kerékjártó [8, pp. 224–226] showed that every periodic homeomorphism of a disk onto itself is a conjugate of either a rotation or a reflection. It was brought to my attention by S. Kinoshita that Kerékjártó in [9] obtains a characterization of those homeomorphisms of S^2 onto itself which are regular; that is, homeomorphisms h such that $\{h^n\}_{n\in I}$ forms an equicontinuous family. It is known [4] that almost periodic homeomorphisms on compact metric spaces satisfy this property, so that our theorem for E^2 would follow from the theorem for S^2.

However, our proof of the main theorem uses Bing's ε-growth technique [6] to obtain an invariant disk, and thus *re*-does a portion of [2], [7], and [9] in a particularly nice way.

Montgomery began a study of almost periodic transformation groups in [13], with the main results for E^3. One very nice theorem states that if G is a one-parameter almost periodic transformation group (a.p.t.g.) of E^3 whose minimal closed invariant sets are one-dimensional, and whose orbits are uniformly bounded, then G is the identity. Our theorem may be regarded as something of an analogue to this theorem for E^2. That is, our theorem shows that if $G = \{h^n\}_{n\in I}$ is an a.p.t.g. of E^2, $h \neq e$, then the orbits are not uniformly bounded.

2. Preliminaries. The definitions used here of the following are as in [4] and [6]: *Relatively dense* subsets of the integers; homeomorphisms *almost periodic at a point*, *pointwise almost periodic* (p.a.p.), and *almost periodic* (a.p.) on the space; *invariant set*; and *minimal set* are defined in [4]. *Property S*, *ε-growth*, and *ε-sequential growth* are defined in [6]. The *orbit* of x in the space X is the set