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ON THE ACTION OF THE DYER-LASHOF ALGEBRA IN $H_*(G)$

IB MADSEN

Let G be the space of homotopy equivalences of S^n for $n \to \infty$. This is an infinite loop space, that is, it has definite deloopings. The first delooping of G is the classifying space for (stable) spherical fibrations.

The (mod. 2) homology ring of an infinite loop space is an algebra over the Dyer-Lashof algebra R of all primary homology operations. The principal result of this paper is the evaluation of the R-action in $H_*(G)$. The R-module $H_*(G)$ determines the R-module $H_*(G/O)$, where G/O is the homogeneous space associated with the infinite orthogonal subgroup of G. Let $\alpha: BSO \to G/O$ be a "solution" of the Adams conjecture in the 2-local category, and let $QH_*(G/O)$ be the R-module of indecomposable elements.

THEOREM. *The induced map $\alpha_*: H_*(BSO) \to Z_2 \otimes_R QH_*(G/O)$ is surjective, in fact $Z_2 \otimes_R QH_*(G/O) \cong QH_*(BSO)$.*

The basic method of the paper is to compare the Boardman-Vogt **[4]** infinite loop space structure on SG, called the *composition-structure* with the *loop-structure* on $Q(S^0) = \lim \Omega^n S^n$. The loop-structure is defined by the identification $Q(S^0) = \Omega^k \lim \Omega^n S^{n+k}$. Let

$$c: R \otimes H_*(SG) \to H_*(SG) \quad \text{and} \quad l: R \otimes H_*(Q(S^0)) \to H_*(Q(S^0))$$

denote the R-actions. The component $Q_0(S^0)$ of $Q(S^0)$ containing the constant map has the homotopy type of SG (the oriented homotopy equivalences) so that $H_*(SG) \cong H_*(Q_0(S^0))$. Roughly, our result on the R-module $H_*(SG)$ is that $c \equiv l$ modulo a certain "length" filtration and modulo totally decomposable elements, that is, decomposable elements of $H_*(SG)$ which are also decomposable in the loop product when considered as elements of $H_*(Q_0(S^0))$. The loop action l was essentially determined in **[10]**. The R-module $H_*(BSG)$ is an easy consequence of the main result.

THEOREM.

$$H_*(BSG) = H_*(BSO) \otimes E\{Q(a,a) \mid a = 1, 2, \cdots\} \otimes P,$$

where P is a (large) *polynomial algebra and $Q(a,a)$ are elements of degree $2a+1$.*