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LOCAL CONNECTEDNESS IN DEVELOPABLE SPACES

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A space is spherically connected if and only if it has an admissible semi-metric d such that d-spheres of radius less than one are connected. It is shown that a developable space is locally connected if and only if it is spherically connected. A semi-metric space is K-semi-metrizable if and only if it admits a semi-metric d such that $d(A, B) > 0$ whenever A and B are disjoint compact sets. It is shown that in the class of locally connected rim compact spaces, the K-semi-metrizable spaces are precisely the developable γ-spaces. An example is given of a locally connected, locally compact K-semi-metrizable Moore space which is not metrizable.

1. Introduction. A topological space is said to be *rim compact* provided that each point has a local basis of open sets which have compact boundaries. A space is *locally connected* provided that each point of the space has a local basis of connected open sets. If R is the set of all rational points of the plane E^2, the $E^2 - R$ is an example of a locally connected, rim compact space which is nowhere locally compact.

If d is a semi-metric for a space X, then d is said to be a *K-semi-metric* provided that $d(A, B) > 0$ whenever A and B are disjoint compact subsets of X. It seems to be unknown whether every regular semi-metrizable space,[1] or even developable space,[2] has a compatible K-semi-metric. We define a topological space X to be *d-spherically connected* provided that X has a compatible semi-metric d such that every d-sphere $S_d(x, e) = \{y: d(x, y) < e\}$ of radius less than one is connected. A space is said to be *spherically connected* provided that it is d-spherically connected for some compatible semi-metric d.

Theorem 5.2 of [3] may be phrased as follows: let X be a rim compact space; if X is d-spherically connected by virtue of a K-semi-metric d, then X is metrizable. Also, P. Zenor has shown that a locally connected rim compact space is metrizable if and only if it

[1] A space X is semi-metrizable provided there exists a nonnegative, real-valued function d on $X \times X$, called a semi-metric, which satisfies the following three conditions: (i) $d(x, y) = d(y, x)$; (ii) $d(x, y) = 0$ iff $x = y$; (iii) for x in X and $A \subset X$, we have $x \in cl(A)$ iff $d(d(x, A)) = \inf\{x, a): a \in A\} = 0$.

[2] A sequence $G_1, G_2, \cdots$ of open covers of a space X is called a development provided that $\{St(x, G_n): n \in Z^+\}$ is a local base at x for each x in X. A space is developable provided it has a development. A regular developable space is called a Moore space.