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LINEAR DIFFERENTIAL SYSTEMS WITH MEASURABLE COEFFICIENTS

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The general homogeneous first order linear differential system is considered. The principal result concerns a representation of the solution space as a direct sum of subspaces such that on each summand upper and lower bounds for the norms of the solutions can be given. The main tool in obtaining this decomposition is the method of fixed points of integral operators.

I. Introduction. Consider the homogeneous linear differential system

$$x'(t) = A(t)x(t) \qquad -\infty < t < \infty \tag{1}$$

where $A(t)$ denotes an nxn complex matrix whose entries are assumed only to be measurable functions of t which are summable on bounded intervals and it is understood that (1) holds almost everywhere. Here x denotes a complex n-vector and for $x = \operatorname{col}(x_1, \cdots, x_n)$ we use $\|x\| = \max_{1 \leq i \leq n} |x|_i$ throughout.

In [4] the second author has shown that when $A(t)$ in (1) is continuous and satisfies a diagonal dominance condition the solution space of (1) admits a type of exponential dichotomy. This result is also discussed in the notes [2, pg. 126-135]. In [1] the first author has established an analogous result for the linear difference equation

$$x(m+1) = A(m+1)x(m) \qquad m = 0, \pm 1, \cdots \tag{2}$$

In §2 we give a more general and improved result for (1), assuming only measurability for $A(t)$, and then use this information to give estimates for upper and lower bounds for solutions to (1). Our estimates are comparative in that they give norm comparisons for solutions at any two values of the variable t. These estimates were obtained by Martin [5] in the continuous case and in a slightly weaker form were announced by the second author in [3]. However, the methods used here are completely different from those used in [4] and [5] and seem more transparent.

In §4 we show, under the additional assumption that $A(t)$ be bounded, that our technique of proof is constructive in that all solutions of (1) bounded on $[0, \infty)$ arise as fixed points of a family of contraction mappings. Finally, we indicate the appropriate analogy with our work concerning (1) for showing that the bounded solutions of (2) arise in a similar manner.