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AN INVERSION OF THE S_2 TRANSFORM FOR GENERALIZED FUNCTIONS

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Define S_2 transform of a member f of a certain space of generalized functions as

$$F(x) = \langle f(t), K(t, x) \rangle$$

where

$$K(t, x) = \begin{cases} \dfrac{\log x/_t}{x - t}, & x \neq t \\ \dfrac{1}{x}, & x = t \end{cases}$$

$(0 < t < \infty, 0 < x < \infty)$.

It is shown that

$$\lim_{n \to \infty} H_{n,x}[F(x)] = f(x)$$

in the weak distributional sence. Here $H_{n,x}$ is a certain linear generalized differential operator.

1. Introduction. Schwartz [6, p. 248] first introduced the Fourier transform of distributions in 1947. Since then, extensions of the classical integral transformations to generalized functions have become of continuing interest. Some references to this effect are [2], [3], [4], [5], [8], [9] and [10]. The Stieltjes and iterated Stieltjes transforms of a function $f(t)$ have been defined respectively as

$$\tilde{f}(u) = \int_0^\infty \frac{f(t)}{u + t} dt , \quad u > 0$$

and

$$\tilde{\tilde{f}}(x) = \int_0^\infty \frac{du}{x + u} \int_0^\infty \frac{f(t)}{u + t} dt , \quad x > 0 .$$

If it is permissible to change the order of integration in the above integral, one gets

$$(1) \qquad \tilde{\tilde{f}}(x) = \int_{0+}^\infty \frac{\log x/t}{x - t} f(t) dt ,$$

where $\log x/t/(x - t)$ is defined by its limiting value $1/x$ at $t = x$. (1) is referred to as the S_2 transform of the function $f(t)$ (see [1, p. 4]). The inversion formula for (1) due to Boas and Widder [1, p. 30] is given by