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UNIQUENESS THEOREMS FOR TAUT SUBMANIFOLDS

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1. Introduction and statements of theorems. Given two closed smooth manifolds, how do you tell if they are diffeomorphic? If you start out with a homotopy equivalence, Browder–Novikoff Theory breaks the problem up into: (1) finding all self-equivalences, (2) finding a normal bordism, and (3) the surgery obstruction on the normal bordism. In applications, however, one may encounter manifolds suspected of being diffeomorphic, where no obvious homotopy equivalence is present.

We describe such a situation: Let $\begin{matrix}\zeta\\ \downarrow\\ M\end{matrix}$ be a simply connected, finite, simplicial complex with linear bundle. Let $K_i^{2n} \xrightarrow{f_i} M$, $i = 0$ or 1, $n \geqq 3$, be normal maps from closed smooth manifolds, i.e. $f_i^*(\zeta) = \nu(K_i)$. Suppose that f_1 and f_2 are normally bordant, f_i is n-connected, and that $B_n(K_0) = B_n(K_1)$. B_n here denotes the n-th Betti number. It follows from Poincaré's Duality and the universal coefficient theorem that K_0 and K_1 have isomorphic integral homology groups, but a map inducing this isomorphism is lacking. However,

THEOREM 1. *If n is odd, K_0 and K_1 are diffeomorphic.*

THEOREM 2. *If n is even, but not 2, and the intersection pairings on*

$$(\operatorname{Ker} f_0\colon H_n(K_0; Z) \to H_n(M; Z))/torsion \qquad and$$
$$(\operatorname{Ker} f_1\colon H_n(K_1; Z) \to H_n(M; Z))/torsion$$

are isometric and nonsingular, then K_0 and K_1 are diffeomorphic.

COROLLARY 1. *If M^{2n+2} is a compact, simply connected, smooth $2n+2$-manifold, n odd, and $K_0^{2n} \xrightarrow{i_0} M^{2n+2}$ and $K_1^{2n} \xrightarrow{i_1} M^{2n+2}$ are n-connected inclusions of closed submanifolds with $i_{0*}[K_0] = i_{1*}[K_1] \in H_{2n}(M^{2n+2}; Z)$, then if $B_n(K_0) = B_n(K_1)$, K_0 is diffeomorphic to K_1.*

COROLLARY 2. *Assume M^{2n+2} is a simply connected smooth $2n+2$-manifold, n even ($n \neq 2$), with $H_n(M; Z) = 0$. If i_0 and i_1 are as above, then if the intersection pairings on $H_n(K_0; Z)/torsion$ and $H_n(K_1; Z)/torsion$ are isometric, K_1 is diffeomorphic to K_2.*

REMARK 1. The above corollaries are specialized by replacing the