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NORM ATTAINING OPERATORS ON $L^1[0, 1]$ AND THE RADON-NIKODÝM PROPERTY

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Let Y be a strictly convex Banach space. Then norm attaining operators mapping $L^1[0, 1]$ to Y are dense in the space of all linear operators from $L^1[0, 1]$ to Y if and only if Y has the Radon-Nikodým property.

Bishop and Phelps [1] have asked the general question—For which Banach spaces X and Y is the collection of norm attaining operators from X to Y dense in the space $B(X, Y)$ of all bounded (linear) operators from X to Y. Lindenstrauss in [8] investigated this question and related this question to existence of extreme points and exposed points in the closed unit ball of X. In the course of his paper Lindenstrauss showed that for some space Y the norm attaining operators in $B(L^1[0, 1], Y)$ are not dense in $B(L^1[0, 1], Y)$ due to the lack of extreme points in the closed unit ball of $L^1[0, 1]$. Left open is the following question: For which Banach spaces Y are the norm attaining operators dense in $B[L^1[0, 1], Y)$? Based on Lindenstrauss's work, one is led to believe that if the closed unit ball of Y has a rich extreme point or exposed point structure, then the norm attaining operators may be dense in $B(L^1[0, 1], Y)$. On the other hand the Radon-Nikodým property is intimately connected with extreme point structure (Rieffel [12], Maynard [10], Huff [6], Davis and Phelps [2], Phelps [11], Huff and Morris [7]). So there is some *prima facie* evidence to support the belief that the norm attaining operators are dense in $B(L^1[0, 1], Y)$ if and only if Y has the Radon-Nikodým property. The purpose of this paper is to verify this for strictly convex Banach spaces Y.

First a few well known results will be collected.

LEMMA A [4, 5]. *If (Ω, Σ, μ) is a finite measure space and $g: \Omega \to Y$ is μ-essentially bounded Bochner integrable function, then*

$$T(f) = Bochner - \int fg d\mu$$

defines a member T of $B(L^1(\mu), Y)$ with $\|T\| = \operatorname{ess\,sup} \|g\|_Y$.

LEMMA B [3]. *Any one of the following statements about Y implies all the others.*

(i) *Y has the Radon-Nikodým property.*

(ii) *If (Ω, Σ, μ) is a finite measure space and $G: \Sigma \to Y$ is a*