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WEIGHTED SIDON SETS

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A weighted generalisation of Sidon sets, W-Sidon sets, is introduced and studied for compact abelian groups. Firstly W-Sidon sets are characterised analogously to Sidon sets and variations of these characterisations shown to lead back to Sidon sets. For the circle group W-Sidon sets are constructed which are not $\Lambda(1)$ and hence not Sidon. The algebra of all W's making a set W-Sidon is investigated and Sidon and p-Sidon sets cast in terms of it. Finally analytic properties of W-Sidon sets are pursued and a necessary condition on the growth of W^2 obtained.

Throughout this paper G denotes a compact abelian Hausdorff topological group and X denotes its (discrete) dual group. Both are written multiplicatively with identities e and 1 respectively.

We write $(L^p(G), \|\cdot\|_p)$ for the Lebesgue space derived from the normalised Haar measure on G and $(C(G), \|\cdot\|_\infty)$ for the space of (complex-valued) functions continuous on G with the supremum norm. However for $\Delta \subseteqq X$ and counting measure on Δ we denote the Lebesgue spaces $(l^p(\Delta), \|\cdot\|_p)$ and use $c_0(\Delta)$ for the subset of $l^\infty(\Delta)$ of functions tending to zero at infinity.

If A and B are sets we write B^A for the set of all functions from A to B; if $f \in B^A$ and $C \subseteqq A$ ($\subset$ is reserved for strict inclusion) we write $f \mid C$ for the restriction of f to C; ξ_A is the characteristic function of A; $\mathfrak{F}(A)$ denotes the set of all finite subsets of A; $\mathfrak{P}(A)$ denotes the power set of A; $\nu(A)$ is the cardinality of A; and we write $\square$ for the empty set.

The sets of complex numbers, real numbers, integers and natural numbers will be written $\mathfrak{C}$, $\mathfrak{R}$, $\mathfrak{Z}$, and $\mathfrak{N}$ respectively and we write $\mathfrak{T}$ for the topological group of unimodular complex numbers. If $c \in \mathfrak{C}$, c denotes the constant function with value c, whose domain will be clear from the context.

For $\Delta \subseteqq X$, $\phi \in \mathfrak{C}^\Delta$ and $A \subseteqq \mathfrak{C}^\Delta$ we write ϕA for $\{\phi\psi: \psi \in A\}$.

We denote the Fourier transform of $f \in L^1(G)$ by $\hat{f}$. If E is a Banach space we write E' for its dual. Let $A(G) = \{f \in C(G): \hat{f} \in l^1(X)\}$ be normed by $\|f\|_A = \|\hat{f}\|_1$ and set the space of pseudomeasures on G, $(PM(G), \|\cdot\|_{PM})$, equal to $A(G)'$ so that it contains $(M(G), \|\cdot\|)$, the space of measures on G. For $\pi \in PM(G)$ we write $\hat{\pi}$ for its Fourier transform and $sp\pi$ for its spectrum, i.e. $\{\chi \in X: \hat{\pi}(\chi) \neq 0\}$. If $E \subseteqq PM(G)$ and $\Delta \subseteqq X$ we let $E_\Delta = \{\pi \in E: sp\pi \subseteqq \Delta\}$ and call its members Δ-spectral pseudomeasures. We also write $E^\wedge$ for $\{\hat{\pi}: \pi \in E\}$.