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NONOSCILLATION THEOREMS FOR DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENT

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The asymptotic behavior of nonoscillatory solutions of a class of nth order nonlinear functional differential equations with deviating argument is investigated. Sufficient conditions are provided which ensure that all nonoscillatory solutions (or all bounded nonoscillatory solutions) of the equations under consideration approach zero as the independent variable tends to infinity. The criteria obtained prove to apply to equations with advanced argument as well as to equations with retarded argument.

1. Introduction. We consider the nth order functional differential equation with deviating argument

$$(1)\quad (r_{n-1}(t)(r_{n-2}(t)(\cdots(r_2(t)(r_1(t)y'(t))')'\cdots)')')' + a(t)f(y(g(t))) = b(t)\ ,$$

where $a(t)$, $b(t)$, $g(t)$, $r_1(t)$, $\cdots$, $r_{n-1}(t)$ are real-valued and continuous on $[\tau, \infty)$ and $f(y)$ is real-valued and continuous on $(-\infty, \infty)$. The following conditions are assumed to hold throughout the paper:

(a) $\lim_{t\to\infty} g(t) = \infty$;

(2) (b) $yf(y) > 0$ for $y \neq 0$;

(c) $r_i(t) > 0$ and $\lim_{t\to\infty} \rho_i(t) = 0$, where

$$\rho_i(t) = \int_t^\infty \frac{\rho_{i-1}(s)}{r_i(s)} ds,\ i = 1, \cdots, n-1,\ (\rho_0(t) \equiv 1)\ .$$

We note that the condition (2c) is satisfied if

$$(3)\qquad \int_\tau^\infty \frac{dt}{r_i(t)} < \infty\ , \qquad i = 1, \cdots, n-1\ .$$

We restrict our consideration to those solutions $y(t)$ of (1) which exist on some ray $[T_y, \infty)$ and satisfy

$$\sup\{|y(t)|: t_0 \leqq t < \infty\} > 0$$

for any $t_0 \in [T_y, \infty)$. Such a solution is said to be oscillatory if it has arbitrarily large zeros; otherwise, it is said to be nonoscillatory.

In the oscillation theory of ordinary differential equations one of the important problems is to find sufficient conditions in order that all (bounded) nonoscillatory solutions of (1) tend to zero as $t \to \infty$. Since the work of Hammett [3] this problem has been the