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OSCILLATION PROPERTIES OF CERTAIN SELF-ADJOINT DIFFERENTIAL EQUATIONS OF THE FOURTH ORDER

GARY D. JONES AND SAMUEL M. RANKIN, III

Assuming oscillation, a connection between the decreasing and increasing solutions of

$$(1) \qquad (ry'')'' = py$$

is established. With this result, it is shown that if $r \equiv 1$ and p positive and monotone the decreasing solution of (1) is essentially unique. It is also shown that if $p > 0$ and $r \equiv 1$ the decreasing solution tends to zero.

It will also be assumed that p and r are positive and continuous and at times continuously differentiable on $[a, +\infty)$. By an oscillatory solution of (1) will be meant a solution $y(x)$ such that there is a sequence $\{x_n\}_{n=1}^{\infty}$ diverging to $+\infty$ such that $y(x_n) = 0$ for every n. Equation (1) will be called oscillatory if it has an oscillatory solution.

Equation (1) has been studied previously by Ahmad [1], Hastings and Lazer [3], Leighton and Nehari [8] and Keener [7].

Hastings and Lazer [3] have shown that if $p > 0$, $r \equiv 1$ and $p' \geqq 0$ then (1) has two linearly independent oscillatory solutions which are bounded on $[a, +\infty)$. They further show that if $\lim_{t\to\infty} p(t) = +\infty$ then all oscillatory solutions tend to zero. Our result will show that there is a nonoscillatory solution which goes to zero "faster" than the oscillatory ones.

Keener [7] shows the existence of a solution y of (1) such that $\operatorname{sgn} y = \operatorname{sgn} y'' \neq \operatorname{sgn} y' = \operatorname{sgn} (ry'')'$. Under the additional hypothesis that $\liminf p(t) \neq 0$ he shows that $y(t) \to 0$ as $t \to \infty$. We will give a condition for $y(t) \to 0$ where $\liminf p(t)$ can be zero.

Ahmad [1] shows that if (1) is nonoscillatory then every solution z of (1) with the properties of y above satisfy $z = cy$ for some constant c.

The following lemmas due to Leighton and Nehari [8] will be basic in our investigation.

LEMMA 1. *If y is a solution of* (1) *with $y(c) \geqq 0$, $y'(c) \geqq 0$, $y''(c) \geqq 0$ and $(r(c)y''(c))' \geqq 0$ but not all zero for $c \geqq a$ then $y(x)$, $y'(x)$, $y''(x)$ and $(r(x)y''(x))'$ are positive for $x > c$.*

LEMMA 2. *If y is a solution of* (1) *with $y(c) \geqq 0$, $y''(c) \geqq 0$, $y'(c) \leqq 0$ and $(r(c)y''(c))' \leqq 0$ but not all zero for $c \geqq a$ then $y(x) > 0$, $y''(x) > 0$, $y'(x) < 0$ and $(r(x)y''(x))' < 0$ for $x \in [a, c)$.*