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KOROVKIN APPROXIMATIONS IN L_p-SPACES

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The main result is a characterization of finite Korovkin sets for positive operators in l_p. It follows that a finite set containing a positive function is a Korovkin set in l_p if and only if it is a Korovkin set in c_0. The methods also show:

PROPOSITION. Let X be a compact subset of R^n. Let K be a subspace of $C(X)$ containing the constants. If K is a Korovkin set in $C(X)$, then K is Korovkin set in $L_p(X)$.

Several related results are also given. For example a question of G. G. Lorentz about the restrictions of Korovkin set in $C(X)$ to a subset $Y \subseteqq X$ is answered.

Let $\mathscr{L}$ be a class of operators on a Banach space E. A subset $K \subseteqq E$ is an *$\mathscr{L}$-Korovkin set* if whenever

(i) $\{L_i\}$ is a bounded sequence in $\mathscr{L}$, and

(ii) $L_i k \rightarrow k$ for each $k \in K$;

we have

(iii) $L_i f \rightarrow f$ for each f in E.

Let $\mathscr{L}^1$ be the class of norm one operators on E. If E is also a lattice, let $\mathscr{L}^+$ denote the positive operators on E; and, $\mathscr{L}^{1,+} = \mathscr{L}^1 \cap \mathscr{L}^+$.

After Korovkin showed that $\{1, x, x^2\}$ is an $\mathscr{L}^+$-Korovkin set in $C[0, 1]$, interest in this field has been in characterizing the Korovkin subsets of the classic Banach spaces.

Papers by Berens and Lorentz [3], Franchetti [8, 9], Krasnosilskii and Lifsic [13], Lorentz [14], Saskin [18], Scheffold [19], and Wulbert [22] identified the various types of Korovkin sets in $C(X)$ spaces. Berens and Lorentz [3] have essentially characterized the $\mathscr{L}^{1,+}$-Korovkin subsets of L_1 spaces (see §3 of this article, also see [Lorentz, 14] and [Wulbert, 22]), and Dzjadyk [7] has shown that $\{1, \sin x, \cos x\}$ is an $\mathscr{L}^+$-Korovkin set in $L_p[0, 2\pi]$. (See also [James, 11], and [Zaricka, 24].)

The results here are related to identifying $\mathscr{L}^+$-Korovkin subsets of L_p-spaces. A sufficient condition is presented that encompasses the known (and the suspected) $\mathscr{L}^+$-Korovkin sets. For example each $\mathscr{L}^+$-Korovkin set in $C[a, b]$ that contains constants is also an $\mathscr{L}^+$-Korovkin set in $L_p[a, b]$. The main result given is a characterization of finite $\mathscr{L}^+$-Korovkin sets in l_p. A consequence of this characterization is that the l_p spaces have the same finite $\mathscr{L}^+$-Korovkin sets. That is, if K is a finite subset of both l_r and l_s, and K contains a