PACIFIC JOURNAL OF MATHEMATICS
Vol. 63, No. 1, 1976

CONCORDANCES OF NONCOMPACT HILBERT CUBE MANIFOLDS

T. A. CHAPMAN

In this paper two exact sequences are established which are useful in computing $\pi_0\mathscr{C}(M)$, the group of isotopy classes of concordances for a noncompact Hilbert cube manifold M. Roughly speaking, this enables one to study the noncompact case in terms of the compact case. The situation is analogous to Siebenmann's description of groups of infinite simple homotopy types in terms of two exact sequences.

1. Introduction. For any space X we will use $\mathscr{C}(X)$ to denote the space of all *concordances* of X. It is the function space, with the compact-open topology, of all homeomorphisms of $I \times X$ onto itself ($I = [0, 1]$) which are the identity on $\{0\} \times X$. We use $\pi_0\mathscr{C}(X)$ to denote the group of all isotopy classes in $\mathscr{C}(X)$, where the group operation is composition. A *Q-manifold* is a separable metric manifold modeled on the Hilbert cube Q, the countable infinite product of closed intervals. In [3] and [4] the author investigated the group $\pi_0\mathscr{C}(M)$ for M a compact Q-manifold. The main result established there was the following: *Let M be a compact Q-manifold which is written as $R \times Q$, where R is a PL n-manifold.* (It follows from [1] that this can always be done.) *Then $\pi_0\mathscr{C}(M)$ is isomorphic to the direct limit of the sequence*

$$\pi_0\mathscr{C}(R) \xrightarrow{(\times id)_*} {}^*\pi_0\mathscr{C}(R \times I) \xrightarrow{(\times id)_*} \pi_0\mathscr{C}(R \times I^2) \longrightarrow \cdots .$$

Fortunately this direct limit has been studied in [8], and as a result we get the following consequences for Q-manifolds.

A. *If M is a compact Q-manifold, then $\pi_0\mathscr{C}(M)$ depends only on the 3-type of M.*

From this we get.

B. *If M and N are homotopy equivalent compact Q-manifolds, then $\pi_0\mathscr{C}(M)$ is isomorphic to $\pi_0\mathscr{C}(N)$.* (See §2 for a proof which uses only infinite-dimensional techniques.)

C. *If M is a compact Q-manifold, then $\pi_0\mathscr{C}(M)$ is trivial iff each component of M is 1-connected.* (This holds in spite of the recently discovered gap in [9].)