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CENTRALIZERS OF TRANSITIVE SEMIGROUP ACTIONS AND ENDOMORPHISMS OF TREES

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A tree is *locally finite* if the interval between any two points is finite. A *local isomorphism* of a tree with itself is a homomorphism which is an isomorphism when restricted to any interval. Two theorems are proved. One characterizes those locally finite trees which have transitive automorphism groups, and those which have transitive local-isomorphism monoids. The other theorem gives necessary and sufficient conditions for a non-injective transformation to be centralized by a transitive permutation group, and necessary and sufficient conditions for it to be centralized by a transitive transformation semigroup. Also, an example is given of a nonlocally-finite tree with transitive automorphism group.

1. Preliminaries and statements of the theorems.

All functions will be written to the right of the argument, and functional composition will read from left to right. If $\alpha: X \to X$ is a function, and $x \in X$, then $x\alpha^{-1}$ denotes the inverse image of x under α. If X is any set, $|X|$ denotes the cardinality of X.

A semigroup S *acts on a set X on the right* if, for every $x \in X$ and $s \in S$, xs denotes an element of X, and

$$(xs)t = x(st) \qquad (x \in X, s \in S). \tag{1}$$

Then X is an *S_r-set* or an *S-operand.*

An action by a semigroup S on X is *transitive* if for every $x, y \in X$, there is an element $s \in S$ such that $xs = y$. A subset G of X *generates* the S-set X if for every $x \in X$ there is $g \in G$ and $s \in S$ such that $gs = x$. Thus an S-set is transitive if and only if every one-element subset of X generates X.

Let X and Y be S-sets. A function $\alpha: X \to Y$ is an *S-homomorphism* (equivariant map) if

$$(x\alpha)s = (xs)\alpha \qquad (x \in X, s \in S). \tag{2}$$

S-endomorphisms and S-automorphisms are defined in the obvious way. It is easy to see that the S-endomorphisms of an S-set X form a semigroup $\mathrm{End}_S X$ and the S-automorphisms form a group $\mathrm{Aut}_S X$.

Let T be a partially ordered set; its order relation, like all those in