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RATIONAL APPROXIMATION OF e^{-x} ON THE POSITIVE REAL AXIS

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In this paper we obtain error bounds to approximations of e^{-x} on $[0;\infty)$ by rational functions having zeros and poles only on the negative real axis.

Our main concern in this paper is the question of approximating e^{-x} on the positive real axis by reciprocals of polynomials and by rational functions, especially by those which have all their zeros and poles on the negative real axis.

NOTATION. Let π_n represent the set of all polynomials of degree $\leqq n$. Let π_n^* represent the set of all polynomials in π_n all of whose zeros are in the left half plane and π_n^{**} represent the set of all polynomials in π_n^* all of whose zeros are real and negative. Similarly let $\rho_n, \rho_n^*, \rho_n^{**}$ represent the sets of rational functions of total degree n whose numerators and denominators are in $\pi_n, \pi_n^*, \pi_n^{**}$ respectively. Let $\| \ \|$ denote $\| \ \|_{L_\infty[0,\infty)}$. Then we define

$$\lambda_{0,n}(f) = \inf_{p\in\pi_n} \left\| f - \frac{1}{p} \right\|,$$

$$\lambda_{0,n}^*(f) = \inf_{p\in\pi_n^*} \left\| f - \frac{1}{p} \right\|,$$

$$\lambda_{0,n}^{**}(f) = \inf_{p\in\pi_n^{**}} \left\| f - \frac{1}{p} \right\|,$$

$$\lambda_n(f) = \inf_{r\in\rho_n} \| f - r \|,$$

$$\lambda_n^*(f) = \inf_{r\in\rho_n^*} \| f - r \|,$$

$$\lambda_n^{**}(f) = \inf_{r\in\rho_n^{**}} \| f - r \|.$$

LEMMA (Newman [1], Theorem 2). *Let $p \in \pi_n^{**}$ where $n \geqq 2$, then*

$$\| e^x - p \|_{L_\infty[0,1]} \geqq (16n+1)^{-1}.$$

We obtain the following results.

(Theorems 1, 2): $(17e^2n)^{-1} \leqq \lambda_{0,n}^{**}(e^{-x}) \leqq (en)^{-1}$, $n \geqq 2$.