PACIFIC JOURNAL OF MATHEMATICS
Vol. 64, No 1, 1976

AN EXAMPLE OF A SIMPLE TRIOD WITH SURJECTIVE SPAN SMALLER THAN SPAN

A. LELEK

The span of a metric space is the least upper bound of numbers α such that, roughly speaking, two points can move over the same portion of the space keeping a distance at least α from each other. The surjective span is obtained if it is required that, in addition, the whole space be covered by each of the moving points. These geometric ideas turn out to be important in continua theory. In the present paper, a simple triod is constructed such that the span of it is strictly greater than the surjective span.

Let X be a connected metric nonempty space. By p_1 and p_2 we denote the standard projections of the product $X \times X$ onto X, that is, $p_1(x, x') = x$ and $p_2(x, x') = x'$ for $(x, x') \in X \times X$. The *surjective span* $\sigma^*(X)$ [resp., the *surjective semispan* $\sigma_0^*(X)$] of X is defined to be the least upper bound of the set of real numbers α with the following property: there exist connected sets $C_\alpha \subset X \times X$ such that $\alpha \leqq \operatorname{dist}(x, x')$ for $(x, x') \in C_\alpha$ and $p_1(C_\alpha) = p_2(C_\alpha) = X$ [resp., $p_1(C_\alpha) = X$]. The *span* $\sigma(X)$ and the *semispan* $\sigma_0(X)$ of X are defined by the formulae:

$$\sigma(X) = \operatorname{Sup}\{\sigma^*(A) : \emptyset \neq A \subset X,\ A \text{ connected}\}, \tag{1}$$

$$\sigma_0(X) = \operatorname{Sup}\{\sigma_0^*(A) : \emptyset \neq A \subset X,\ A \text{ connected}\}. \tag{2}$$

It follows directly from the definitions that the following inequalities hold:

$$0 \leqq \sigma^*(X) \leqq \sigma(X) \leqq \sigma_0(X) \leqq \operatorname{diam} X, \tag{3}$$

$$0 \leqq \sigma^*(X) \leqq \sigma_0^*(X) \leqq \sigma_0(X) \leqq \operatorname{diam} X, \tag{4}$$

$$\sigma(A) \leqq \sigma(X), \qquad \sigma_0(A) \leqq \sigma_0(X) \qquad (A \subset X). \tag{5}$$

It is not difficult to check that the above definition of the span, formula (1), is equivalent to the definition given in **[5]**. Continua of surjective span zero were defined in **[10]**. For each arc, as well as for each arc-like continuum, all these four quantities are equal to zero (cf. **[8]**, Propositions 1.3 and 2.1). Nevertheless, they are quite useful in the theory of tree-like continua (see **[2]**, **[3]**, **[6]**, **[7]** and **[8]**). From this