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NON-HAUSDORFF MULTIFUNCTION GENERALIZATION OF THE KELLEY-MORSE ASCOLI THEOREM

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The paper generalizes the Kelley-Morse theorem to continuous point-compact multifunction context. The generalization, which is non-Hausdorff, contains the Ascoli theorem for continuous functions on a k_3-space by the authors and the known multifunction Ascoli theorems of Mancuso and of Smithson.

1. Introduction. The Kelley-Morse theorem [**3**, p. 236] is central among the topological Ascoli theorems for continuous functions on a k-space. It generalizes to the k_3-space theorem of [**1**], which contains all known Ascoli theorems for k-spaces or k_3-spaces.

Obviously a multifunction generalization depends on a multifunction extension of "even continuity". One such extension is that of Lin and Rose [**5**], but this was not applied in Kelley-Morse context. Another which was so applied [**7**, p. 24] is two-fold and leads to a two-fold multifunction Kelley-Morse theorem which, however, does not contain the Mancuso theorem [**6**, p. 470], nor the Smithson theorem [**9**, p. 259]. This paper gives a natural multifunction extension of the definition and leads to a multifunction theorem containing all the above-mentioned theorems.

2. Tychonoff sets. Let X and Y be nonempty sets. A *multifunction* is a point to set correspondence $f: X \to Y$ such that, for all $x \in X$, fx is a nonempty subset of Y. For $A \subseteq X$, $B \subseteq Y$ it is customary to write $f(A) = \bigcup_{x \in A} fx$, $f^-(B) = \{x: x \in X \text{ and } fx \cap B \neq \emptyset\}$ and $f^+(B) = \{x: x \in X \text{ and } fx \subseteq B\}$. If Y is a topological space, a multifunction $f: X \to Y$ is *point-compact* if fx is compact for all $x \in X$.

Let $\{Y_x\}_{x \in X}$ be a family of nonempty sets. The *m-product* $P\{Y_x: x \in X\}$ of the Y_x is the set of all multifunctions $f: X \to \bigcup_{x \in X} Y_x$ such that $fx \subseteq Y_x$ for all $x \in X$. In the case $Y_x = Y$ for all $x \in X$, the m-product of the Y_x, denoted Y^{mX}, is the set of all multifunctions on X to Y. In particular, if Y is a topological space, the symbol $(Y^{mX})_0$ will denote the set of all point-compact members of Y^{mX}. For $x \in X$, the *x-projection* $\text{pr}_x: P\{Y_x: x \in X\} \to Y_x$ is the multifunction defined by $\text{pr}_x f = fx$. If the Y_x are topological spaces, the *pointwise topology* τ_p on $P\{Y_x: x \in X\}$ is defined to be the topology having as open subbase the sets of the forms $\text{pr}_x^-(U_x)$, $\text{pr}_x^+(U_x)$, where U_x is open in Y_x, $x \in X$.

For $F \subseteq Y^{mX}$, $x \in X$, we write $F[x] = \bigcup_{f \in F} fx$. Let Y be a topological space. A subset F of Y^{mX} is *pointwise bounded* if $F[x]$ has compact