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THE DUAL OF A SPACE WITH THE RADON–NIKODYM PROPERTY

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Two characterizations of a Banach space with the Radon–Nikodym property are proved here. The first shows its equivalence with a condition on the dual space which is somewhat weaker than that of being an Asplund space. This leads to a second characterization by a renorming property.

A convex function f on a Banach space X will be assumed to take its values in $(-\infty, +\infty]$ and to be finite at some point. The *domain of continuity* of f is the convex open set of all points at which f is finite and continuous. The space X is called an *Asplund space* if each convex function on X is Fréchet differentiable on a dense G_δ subset of its domain of continuity. If X is the dual of a Banach space Y, then it will be called a *weak*-Asplund space* if each weak* lower semi-continuous (w^*-lsc) convex function on X is Fréchet differentiable on a dense G_δ subset of its domain of continuity. The terms "G_δ" and "domain of continuity" here still refer to the norm topology on X. Thus a dual space which is an Asplund space is also a weak*-Asplund space. A Banach space may be said to have the *Radon–Nikodym property* (RNP) if each closed bounded convex subset is the closed convex hull of its strongly exposed points **[6]**. A point x in a set C is said to be *strongly exposed* by a linear functional y if the supremum of y over C is finite and attained at x and $\|x_i - x\| \to 0$ whenever $\{x_i\}$ is a sequence in C for which $y(x_i) \to y(x)$.

Using the same method as in **[3]**, we characterize the dual of a space with the RNP by its differentiability properties. This allows us to give an alternate proof of a result of Huff and Morris **[4]** concerning the density of strongly exposing functionals and to observe that weak*-Asplund spaces enjoy some of the permanence properties that Asplund spaces do.

THEOREM 1. *A Banach space X has the* RNP *if and only if X^* is a weak*-Asplund space.*

Proof. Assume X has the RNP and let f be a w^*-lsc convex function on X^* with nonempty domain of continuity D. Choose any point $w \in D$ and an $\varepsilon > 0$ so that f is bounded on $N = \{y : \|y - w\| \leqq \epsilon\}$ and $N \subseteq D$. We use the dual norm so that N is weak* closed. Define g