PACIFIC JOURNAL OF MATHEMATICS
Vol. 64, No. 1, 1976

NONOSCILLATION THEORY OF ELLIPTIC EQUATIONS OF ORDER $2n$

W. ALLEGRETTO

Several nonoscillation theorems are obtained for elliptic equations of order $2n$. These results extend several well known nonoscillation theorems for elliptic equations of order 2 and 4, and for ordinary differential equations of higher order.

Introduction. Several authors have considered the problem of establishing oscillation and nonoscillation criteria for elliptic equations. We refer the reader to the books by C. A. Swanson **[15]** and K. Kreith **[8]** where extensive bibliographies can be found.

Most of the interest has so far centered on second order equations, with some results also established for fourth order equations. In this paper we establish several nonoscillation theorems for elliptic equations of order $2n$. These theorems extend in particular, results of Swanson **[14]**, Piepenbrink **[12]**, Headley and Swanson **[5]** and Yoshida **[16]**.

Our proofs make extensive use of variational arguments, of extended Sobolev-type inequalities and of estimates on quadratic forms associated with elliptic equations.

The first part of the paper discusses some preliminary comparison theorems and lower estimates on quadratic forms. The second part deals with the nonoscillation of operations defined in subdomains of E^m for $m \neq 2$. In the next part, some results are established for operations defined in subdomains of E^2. The final part deals with extensions to more general cases.

Definitions and notations. Let Ω be an unbounded domain of m-dimensional Euclidean space E^m. Without loss of generality, we may assume $0 \notin \bar{\Omega}$. Points of E^m are denoted by $x = (x_1, \cdots, x_m)$ and differentiation with respect to x_i by D_i, $i = 1, \cdots, m$. Let L be the differential expression given by:

$$Lu = (-1)^n \sum_{|\alpha|=|\beta|=n} D^\alpha (a_{\alpha\beta} D^\beta u) - a_{00} u, \qquad a_{\alpha\beta} = a_{\beta\alpha}$$

whose coefficients are real defined in Ω and sufficiently regular so that all derivatives involved in L exist and are at least continuous in the closure of $\Omega - R$ for some sphere R. As is usual, we set $D^\alpha u = D_1^{\alpha(1)} \cdots D_m^{\alpha(m)} u$,