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GAUSS SUMS AND INTEGRAL QUADRATIC FORMS OVER LOCAL FIELDS OF CHARACTERISTIC 2

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The theory of Gauss sums is developed for integral quadratic forms over a local field of characteristic 2, and Gauss sums are used to characterize these forms. For a character χ and an integral lattice L, the Gauss sum $\chi(L)$ is either zero, a nonnegative power of two, or the negative of a positive power of two. Gauss sums alone characterize the integral equivalence classes for modular lattices. For arbitrary lattices, other invariants are required.

The classification given in this paper is an alternate to the one by C.-H. Sah [6]. The notation and terminology of [6] is used except when stated to the contrary. O. T. O'Meara [4] used Gauss sums to characterize local integral quadratic forms over a field of characteristic not 2, and R. Jacobowitz [3] classified hermitian forms over the integers of a local field of characteristic not 2 by Gauss sums. When needed, results from these papers are referred to when the proofs hold for the characteristic 2 case.

After a few preliminaries, we introduce Gauss sums and prove some results for Gauss sums of lines and planes that will in turn be used to study more complicated lattices. In Theorem 5.4 we show that Gauss sums alone are sufficient to characterize modular lattices, and in Theorem 4.2 we show that for nondefective lattices only a finite number of Gauss sums need be considered.

1. Preliminaries.

Throughout this paper k denotes a local field of characteristic 2 with fixed prime element π, ring of integers $\mathfrak{o}$, and residue class field of order 2^f. We let Ω denote a complete set of representatives for the residue class field. We refer the reader to [6] for a discussion of the Arf invariant ΔV for a quadratic space V and the additive group $\mathscr{P}\Omega$. As in [6] we let $\{0, \lambda\}$ be a fixed set of representatives of $\Omega/\mathscr{P}\Omega$. The letter e always denotes a unit of k. For a nonnegative integer s, H_s and H_s' denote s-hyperbolic lattices.

Let L be a lattice. $K(L) = \{x \in L \mid \langle x, y\rangle = 0 \text{ for all } y \in L\}$. If $K(L) = 0$, then L is nondefective. Otherwise, it is defective. We assume that if $x \in K(L)$ and $Q(x) = 0$, then $x = 0$.

We now state some lemmas and definitions from [6] in the form in which they are used in this paper.