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REARRANGEMENTS OF FUNCTIONS ON THE RING OF INTEGERS OF A p-SERIES FIELD

BENJAMIN B. WELLS, JR.

We show that every continuous function on the ring of integers of a p-series field has a rearrangement that has absolutely convergent Fourier series.

I. Introduction. Let p be a rational prime fixed throughout. K will denote the p-series field of formal Laurent series in one variable with finite principal part and with coefficients in $GF(p)$, Thus, an element $x \in K$ has representation as

$$x = \sum a_j \mathfrak{p}^j \qquad (a_j = 0, 1, \cdots, p-1)$$

and $a_j = 0$ for j sufficiently small. Addition and multiplication are defined by the usual formal sums and products of Laurent series.

The field K is topologized by taking as a basis the sets

$$V_{x,k} = \{\sum b_j \mathfrak{p}^j \colon b_j = a_j,\ j < k\}$$

where $x = \sum a_j \mathfrak{p}^j$. With this topology, K is locally compact, totally disconnected and nondiscrete.

The ring of integers $\mathfrak{O} = \{x \colon x = \sum_{j=0}^{\infty} a_j \mathfrak{p}^j\}$ is the unique maximal compact subring of K. Let dx denote Haar measure on K derived from the additive structure and normalized so that $\mathfrak{O}$ has measure 1.

As a locally compact abelian group, $\mathfrak{O}$ has a Pontryagin dual $\hat{\mathfrak{O}}$ that may be identified with $K/\mathfrak{O}$. We choose the representatives of the form

$$\sum_{-1}^{-\nu} r_j \mathfrak{p}^j \qquad (r_j = 0, 1, \cdots, p-1)$$

and use the lexicographic ordering to match the characters χ_t to the nonnegative integers. Of course, if χ is a continuous unitary character of K^+, then $\chi(x)$ is a pth root of unity for all $x \in K$.

If f is an integrable function on $\mathfrak{O}$, its Fourier coefficients are given by

$$\hat{f}(t) = \int_{\mathfrak{O}} f(x)\bar{\chi}_t(x)dx \qquad (t = 0, 1, \cdots)\,.$$

We define the class $A(\mathfrak{O})$ of continuous complex-valued functions on $\mathfrak{O}$ as those functions f for which the quantity

$$\sum_{t=0}^{\infty} |\hat{f}(t)|$$