PACIFIC JOURNAL OF MATHEMATICS
Vol. 65, No. 1, 1976

TOPOLOGIES FOR PROBABILISTIC METRIC SPACES

ROBERT M. TARDIFF

Profile functions are used to construct a family of closure operators (in the sense of Čech) on a probabilistic metric space. Relationships among the various closure operators are considered, and products and quotients of probabilistic metric spaces are reexamined in light of this new topological structure.

0. **Introduction.** In their original paper [9], B. Schweizer and A. Sklar introduced a neighborhood structure for a probabilistic metric (PM) space, which, under suitable conditions, is metrizable [11]. However, the usefulness of this neighborhood structure is limited to those spaces in which, for every $\varepsilon > 0$, there exist pairs of distinct points which have probabilities greater than $1 - \varepsilon$ assigned to the event that the distance between them is less than ε. For example, C-spaces [10] do not have this property, with the result that the neighborhood structure of Schweizer and Sklar is discrete.

In [13] E. Thorp and in [6] R. Fritsche tried to overcome this difficulty, but, in so doing, each imposed a neighborhood structure on the PM space which, in general, failed to satisfy the following fundamental neighborhood axiom: If N_1 and N_2 are neighborhoods of a point p, then there is a neighborhood N_3 of p such that N_3 is contained in the intersection of N_1 and N_2. Thus, each of their neighborhood structures did not yield a topology on the PM space, nor even a closure operator in the sense of Čech [2].

In this paper we use the profile functions introduced by Fritsche in [6] to construct a family of neighborhood structures for a PM space. With these neighborhood structures the difficulties incurred by Schweizer and Sklar are easily overcome. Furthermore, we show that for each profile function, the associated neighborhood structure satisfies the aforementioned neighborhood axiom, and hence, yields a closure operator on the PM space in the sense of Čech, and we determine sufficient conditions for this closure operator to be a closure operator in the sense of Kuratowski. We also study the relationships among the neighborhood structures determined by different profile functions and discuss the separation axioms in this context.

Next, we extend the work of R.J. Egbert [3] on products of PM spaces, the probabilistic diameter, and the probabilistic Hausdorff metric in two directions: First, we redefine these concepts in terms